Journal of Advances in Mathematics and Computer Science

27(6): 1-12, 2018; Article no.JAMCS.41220
ISSN: 2456-9968
(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

A Family of Continuous Blended Block A-Stable Second Derivative Linear Multistep Methods for the solutions of Stiff System of Ordinary Differential Equations

Omagwu Samson[1*], J. P. Chollom[2], G. M. Kumleng[2] and Muhamad Shakur Ndayawo[1]

[1]Department of Mathematics and Statistics, Kaduna Polytechnic, Kaduna.
[2]Department of Mathematics University of Jos, Jos, Nigeria.

Authors' contributions

This work was carried out in collaboration between all authors. Author OS derived the methods and did the implementation of the methods. Author JPC did the convergence analysis. Author GMK plotted the region of absolute stability. Author MSN did the literature review. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2018/41220
Editor(s):
(1) Dr. Morteza Seddighin, Professor, Indiana University East Richmond, USA.
(2) Dr. Paul Bracken, Professor, Department of Mathematics, The University of Texas-Pan American, Edinburg, USA.
Reviewers:
(1) Aliyu Bhar Kisabo, Nigeria.
(2) Wale Okunuga, University of Lagos, Nigeria.
(3) Ali Akgül, Siirt University, Turkey.
Complete Peer review History: http://www.sciencedomain.org/review-history/25323

Received: 2nd April 2018
Accepted: 19th June 2018
Published: 29th June 2018

Original Research Article

Abstract

In this paper, the formulation of the block second derivative Blended Linear Multistep methods for step numbers k=5,6 and7 was considered. We present a new family of blended block A-stable second derivative linear multistep methods of order p=k+2 for step numbers k=5,6 and 7 for the solution of stiff initial value problems. The newly constructed blended block methods are all A-stable, consistent, zero-stable and as such convergent. Numerical examples are considered to show the performance of the new methods.

Keywords: A-stable; blended linear multistep methods; second derivative and stiff ODEs.

*Corresponding author: E-mail: samsonomagwu@yahoo.com;

1 Introduction

Numerical solution of real life modelled problems in areas like engineering, science and social sciences often lead to stiff systems of first order Ordinary Differential Equations (ODEs) of the form: Some of these real-life modelled equations do not have analytic solutions as such the need for good numerical methods to approximate their solutions.

In this paper, we are a concern with the numerical solution of the stiff initial value problem (1) using the second derivative linear multistep.

$$y'(x) = f(x, y(x)), \quad y(x_0) = y_0 \tag{1}$$

The solutions of (1) using the known analytical methods are not always easy, and some cases cannot even be solved at all using these methods. The Robertson and Van der pol's equations are good examples of (1). Numerical analysts have developed methods for the numerical solutions of both stiff and non-stiff problems. Researchers like [1-3], the second derivative methods of [4], Enright constructed the blended linear multistep methods of order two for solving (1) directly, [5] constructed a 4-point block method for direct integration of (1). In this paper, a new continuous linear multistep method, in which the approximate solution is the combination of power series and an exponential function. Other authors who have done considerable work on the numerical solution of (1) for the first derivative case Include [6-9]. The second derivative has been studied for the solution of (1) by some authors like [4,10-15], and so on. Such methods either relax the condition to obtain A $-$ stable methods or incorporate off-step points to improve the stability of the methods.

The objectives of paper are to develop a class of A-stable blended block linear multistep methods of high order using the collocation approach as in Enright and to implement these methods in block form as numerical integrators to eliminate the use of starting values.

2 Formulations of the Methods

In this second a k - step the second derivative of the form

$$y(x) = \sum_{j=0}^{t-1} \alpha_j\,(x) y_{n+j} + h \sum_{j=0}^{t-1} \beta_j\,(x) f_{n+j} + h^2 \sum_{j=0}^{t-1} \lambda_j\,(x) y''_{n+j} \tag{2}$$

where α_j, β_j and γ_j are parameters to be determined and using the collocation approach of [16], [17] to construct we obtain the continuous formulation of (2) for k = 5 as

$$\bar{y}\,(x_{n+5}) = \alpha_4(x) y_{n+4} + h[\beta_0(x) f_n + \beta_1(x) f_{n+1} + \beta_2(x) f_{n+2} + \beta_3(x) f_{n+3} + \beta_4(x) f_{n+4} + \beta_5(x) f_{n+5}] + h^2 \lambda_5(x) y''_{n+5} \tag{3}$$

and the D matrix as

$$D = \begin{bmatrix} 1 & (x_n + 4\mathrm{h}) & (x_n + 4\mathrm{h})^2 & \cdots & (x_n + 4\mathrm{h})^6 \\ 0 & 1 & 2\mathrm{x_n} & \dots & 6\mathrm{x_n}^5 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 2(x_n + 5\mathrm{h}) & \dots & 6(x_n + 5\mathrm{h})^5 \\ 0 & 0 & 2 & \cdots & 30(x_n + 5\mathrm{h})^4 \end{bmatrix} \tag{4}$$

The continuous scheme is given as

$$\bar{y}(\tau + x_n) = y_{n+4} + \left(\tau - \frac{149\tau^2}{120h} + \frac{1399\tau^3}{1800h^2} - \frac{13\tau^4}{48h^3} + \frac{4\tau^5}{75h^4} - \frac{\tau^6}{180h^5} - \frac{\tau^7}{4200h^6} - \frac{158h}{525}\right) f_n$$

$$+\left(\frac{25\tau^2}{8h}-\frac{445\tau^3}{144h^2}+\frac{590\tau^4}{384h^3}-\frac{47\tau^5}{160h^4}+\frac{19\tau^6}{576h^5}-\frac{\tau^7}{672h^6}-\frac{52h}{35}\right)f_{n+1}$$
$$+\left(\frac{-25\tau^2}{6h}+\frac{595\tau^3}{108h^2}-\frac{67\tau^4}{24h^3}+\frac{31\tau^5}{45h^4}-\frac{\tau^6}{12h^5}+\frac{\tau^7}{252h^6}-\frac{344h}{945}\right)f_{n+2}$$
$$+\left(\frac{25\tau^2}{6h}-\frac{215\tau^3}{36h^2}+\frac{323\tau^4}{72h^3}-\frac{109\tau^5}{120h^4}+\frac{17\tau^6}{144h^5}-\frac{\tau^7}{168h^6}-\frac{176h}{105}\right)f_{n+3}$$
$$+\left(\frac{-25\tau^2}{6h}+\frac{335\tau^3}{72h^2}-\frac{133\tau^4}{48h^3}+\frac{4\tau^5}{5h^4}-\frac{\tau^6}{9h^5}+\frac{\tau^7}{168h^6}-\frac{2h}{35}\right)f_{n+4}$$
$$+\left(\frac{149\tau^2}{120h}-\frac{20269\tau^3}{10800h^2}+\frac{439\tau^4}{384h^3}-\frac{2449\tau^5}{7200h^4}+\frac{47\tau^6}{960h^5}+\frac{137\tau^7}{50400h^6}-\frac{548h}{4725}\right)f_{n+5}$$
$$+\left(\frac{-\tau^2}{2}+\frac{137\tau^3}{180h}-\frac{15\tau^4}{32h^2}+\frac{17\tau^5}{120h^3}-\frac{\tau^6}{48h^4}+\frac{\tau^7}{840h^5}+\frac{16h^2}{315}\right)g_{n+5}$$

$$g_{n+5}=y''_{n+5} \tag{5}$$

Evaluating the continuous scheme (3) at $\tau = 0, h, 2h, 3h, 5h$ yields the main and additional method which are combine as blended block methods of Five-Step as a global solution for (1)

$$\boldsymbol{y_n = y_{n+4} - \frac{158}{252}hf_n - \frac{52}{32}hf_{n+1} - \frac{344}{945}hf_{n+2} - \frac{176}{105}hf_{n+3} - \frac{2}{35}hf_{n+4} - \frac{548}{4725}hf_{n+5} - \frac{16}{315}h^2y''_{n+5}}$$

$$\boldsymbol{y_{n+2} = y_{n+4} - \frac{1}{630}hf_n - \frac{53}{2520}hf_{n+1} - \frac{382}{945}hf_{n+2} - \frac{773}{630}hf_{n+3} - \frac{263}{630}hf_{n+4} + \frac{221}{7560}hf_{n+5} - \frac{1}{126}h^2y''_{n+5}}$$

$$\boldsymbol{y_{n+3} = y_{n+4} - \frac{19}{8400}hf_n - \frac{89}{4480}hf_{n+1} + \frac{677}{7560}hf_{n+2} - \frac{1933}{3360}hf_{n+3} - \frac{323}{560}hf_{n+4} + \frac{48467}{604800}hf_{n+5} - \frac{271}{10080}h^2y''_{n+5}}$$

$$\boldsymbol{y_{n+5} = y_{n+4} + \frac{41}{25200}hf_n - \frac{529}{40320}hf_{n+1} + \frac{373}{7560}hf_{n+2} - \frac{1271}{10080}hf_{n+3} + \frac{2837}{5040}hf_{n+4} + \frac{317731}{604800}hf_{n+5} - \frac{863}{10080}h^2y''_{n+5}} \tag{6}$$

Similarly, the continuous formulation for K=6 and 7 were obtained using the general form of the Blended Block Linear Multistep method for k=6 as:

$$\bar{y}(x_{n+6}) = \alpha_5(x)y_{n+5} + h[\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3} + \beta_4(x)f_{n+4} + \beta_5(x)f_{n+5} + \beta_6(x)f_{n+6}] + h^2\lambda_6(x)y''_{n+6} \tag{7}$$

and the D matrix as

$$D = \begin{bmatrix} 1 & (x_n+5h) & (x_n+5h)^2 & \cdots & (x_n+5h)^7 \\ 0 & 1 & 2x_n & \ldots & 7{x_n}^6 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 2(x_n+6h) & \ldots & 7(x_n+6h)^6 \\ 0 & 0 & 2 & \cdots & 42(x_n+6h)^5 \end{bmatrix} \tag{8}$$

The continuous scheme is given as

$$\bar{y}(\tau+x_n) = y_{n+5} + \left(\tau - \frac{157\tau^2}{120h} + \frac{959\tau^3}{1080h^2} - \frac{3017\tau^4}{8640h^3} + \frac{119\tau^5}{1440h^4} - \frac{301\tau^6}{25920h^5} + \frac{\tau^7}{1120h^6} + \frac{\tau^8}{4800h^7} - \frac{42655h}{145152}\right)f_n$$
$$+\left(\frac{18\tau^2}{5h}-\frac{97\tau^3}{25h^2}+\frac{377\tau^4}{200h^3}-\frac{151\tau^5}{300h^4}+\frac{11\tau^6}{144h^5}-\frac{`13\tau^7}{2100h^6}+\frac{\tau^8}{4800h^7}-\frac{6185h}{4032}\right)f_{n+1}$$

$$\left(\frac{-45\tau^2}{8h}-\frac{127\tau^3}{16h^2}-\frac{289\tau^4}{64h^3}-\frac{1283\tau^5}{960h^4}-\frac{251\tau^6}{1152h^5}+\frac{25\tau^7}{1344h^6}-\frac{\tau^8}{1536h^7}-\frac{7375h}{32256}\right)f_{n+2}$$
$$+\left(\frac{20\tau^2}{3h}-\frac{274\tau^3}{27h^2}+\frac{685\tau^4}{108h^3}-\frac{61\tau^5}{40h^4}+\frac{229\tau^6}{648h^5}-\frac{2\tau^7}{63h^6}+\frac{\tau^8}{864h^7}-\frac{33625h}{18144}\right)f_{n+3}$$
$$+\left(\frac{-45\tau^2}{8h}+\frac{71\tau^3}{8h^2}-\frac{373\tau^4}{64h^3}+\frac{949\tau^5}{480h^4}-\frac{209\tau^6}{576h^5}+\frac{23\tau^7}{672h^6}-\frac{\tau^8}{768h^7}-\frac{5875h}{16128}\right)f_{n+4}$$
$$+\left(\frac{18\tau^2}{5h}-\frac{29\tau^3}{5h^2}+\frac{157\tau^4}{40h^3}-\frac{83\tau^5}{60h^4}+\frac{191\tau^6}{720h^5}-\frac{11\tau^7}{420h^6}-\frac{\tau^8}{960h^7}-\frac{3805h}{4032}\right)f_{n+5}$$
$$+\left(\frac{-157\tau^2}{120h}+\frac{22979\tau^3}{10800h^2}-\frac{21019\tau^4}{14400h^3}+\frac{7543\tau^5}{14400h^4}-\frac{355\tau^6}{3456h^5}+\frac{1049\tau^7}{100800h^6}-\frac{49\tau^8}{115200h^7}-\frac{21065h}{96768}\right)f_{n+6}$$
$$+\left(\frac{\tau^2}{2h}-\frac{49\tau^3}{60h}+\frac{203\tau^4}{360h^2}-\frac{49\tau^5}{240h^3}+\frac{35\tau^6}{864h^4}-\frac{\tau^7}{240h^5}+\frac{\tau^8}{5760h^7}-\frac{275h}{3456}\right)g_{n+6}$$

$$g_{n+6}=y''_{n+6} \tag{9}$$

the general form of the Blended Block Linear Multistep method for k=7 as:

$$\bar{y}\,(x_{n+7})=\alpha_6(x)y_{n+6}+h[\beta_0(x)f_n+\beta_1(x)f_{n+1}+\beta_2(x)f_{n+2}+\beta_3(x)f_{n+3}+\beta_4(x)f_{n+4}+\beta_5(x)f_{n+5}+\beta_6(x)f_{n+6}+\beta_7(x)f_{n+7}]+h^2\lambda_7(x)y''_{n+7} \tag{10}$$

and the D matrix as

$$D=\begin{bmatrix}1 & (x_n+6\text{h}) & (x_n+6\text{h})^2 & \cdots & (x_n+6\text{h})^8\\ 0 & 1 & 2\text{x}_\text{n} & \cdots & 11\text{x}_\text{n}{}^7\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 1 & 2(x_n+7\text{h}) & \cdots & 8(x_n+7\text{h})^7\\ 0 & 0 & 2 & \cdots & 56(x_n+7\text{h})^6\end{bmatrix} \tag{11}$$

The continuous scheme is given as

$$\bar{y}(\tau+x_n)=y_{n+6}+\Big(\tau-\frac{1159\tau^2}{840h}+\frac{1531\tau^3}{1512h^2}-\frac{3839\tau^4}{8640h^3}+\frac{2647\tau^5}{21600h^4}-\frac{139\tau^6}{6480h^5}+\frac{\tau^7}{432h^6}-\frac{17\tau^8}{120960h^7}-\frac{17\tau^9}{1272160h^8}-199h700fn$$
$$+\left(\frac{21\tau^2}{5h}-\frac{739\tau^3}{150h^2}-\frac{3221\tau^4}{1200h^3}-\frac{7547\tau^5}{9000h^4}+\frac{299\tau^6}{1440h^5}-\frac{457\tau^7}{25200h^6}+\frac{11\tau^8}{9600h^7}-\frac{\tau^9}{32400h^8}-\frac{1413h}{875}\right)f_{n+1}$$
$$+\left(\frac{-63\tau^2}{8h}+\frac{949\tau^3}{80h^2}-\frac{601\tau^4}{80h^3}+\frac{12449\tau^5}{4800h^4}-\frac{19\tau^6}{36h^5}+\frac{71\tau^7}{1120h^6}-\frac{\tau^8}{240h^7}+\frac{\tau^9}{8640h^8}+\frac{27h}{350}\right)f_{n+2}$$
$$+\left(\frac{35\tau^2}{3h}-\frac{10194\tau^3}{54h^2}+\frac{5617\tau^4}{432h^3}-\frac{5213\tau^5}{1080h^4}+\frac{2701\tau^6}{2592h^5}-\frac{397\tau^7}{3024h^6}-\frac{31\tau^8}{3456h^7}+\frac{\tau^9}{3888h^8}-\frac{89h}{35}\right)f_{n+3}$$
$$+\left(\frac{-105\tau^2}{8h}+\frac{527\tau^3}{24h^2}-\frac{3037\tau^4}{192h^3}+\frac{8881\tau^5}{1440h^4}-\frac{67\tau^6}{48h^5}+\frac{185\tau^7}{1008h^6}-\frac{5\tau^8}{384h^7}+\frac{\tau^9}{5292h^8}-\frac{99h}{140}\right)f_{n+4}$$
$$+\left(\frac{63\tau^2}{5h}-\frac{43\tau^3}{2h^2}+\frac{1273\tau^4}{80h^3}-\frac{3847\tau^5}{600h^4}+\frac{2167\tau^6}{1440h^5}-\frac{23\tau^7}{112h^6}-\frac{29\tau^8}{1920h^7}-\frac{\tau^9}{2160h^8}-\frac{459h}{175}\right)f_{n+5}$$
$$+\left(\frac{679\tau^2}{120h}+\frac{104933\tau^3}{10800h^2}-\frac{26113\tau^4}{3600h^3}+\frac{212701\tau^5}{72000h^4}-\frac{379\tau^6}{540h^5}+\frac{4889\tau^7}{50400h^6}+\frac{13\tau^8}{1800h^7}-\frac{29\tau^9}{129600h^8}-\frac{401h}{1750}\right)f_{n+6}$$

$$+\left(\frac{3\tau^2}{7h}+\frac{157\tau^3}{210h^2}-\frac{137\tau^4}{240h^3}+\frac{431\tau^5}{1800h^4}-\frac{17\tau^6}{288h^5}+\frac{43\tau^7}{5040h^6}-\frac{3\tau^8}{4480h^7}-\frac{\tau^9}{45360h^8}+\frac{9h}{175}\right)f_{n+7}$$
$$+\left(\frac{7\tau^2}{2h}-\frac{121\tau^3}{20h}+\frac{3283\tau^4}{720h^2}-\frac{6769\tau^5}{3600h^3}+\frac{49\tau^6}{108h^4}-\frac{23\tau^7}{360h^5}+\frac{7\tau^8}{1440h^7}-\frac{\tau^9}{6480h^8}-\frac{9h}{25}\right)g_{n+7}$$

$$g_{n+7}=y''_{n+7} \tag{12}$$

these continuous formulations are evaluated at $\tau=0,h,2h,3h,4h,6h$ and $\tau=0,h,2h,3h,4h,5h,7h$ for k=6 and 7 respectively to yield the following blended block methods.

$$y_n=y_{n+5}-\frac{42655}{145152}hf_n-\frac{6185}{4032}hf_{n+1}-\frac{7375}{32256}hf_{n+2}-\frac{33625}{18144}hf_{n+3}-\frac{5875}{16128}hf_{n+4}-\frac{3805}{4032}hf_{n+5}+\frac{21065}{96768}hf_{n+6}-\frac{275}{3456}h^2y''_{n+6}$$
$$y_{n+1}=y_{n+5}+\frac{8}{945}hf_n-\frac{38}{105}hf_{n+1}-\frac{136}{105}hf_{n+2}-\frac{664}{945}hf_{n+3}-\frac{136}{105}hf_{n+4}-\frac{38}{105}hf_{n+5}+\frac{8}{945}hf_{n+6}$$
$$y_{n+2}=y_{n+5}-\frac{13}{5376}hf_n+\frac{327}{11200}hf_{n+1}-\frac{7923}{17920}hf_{n+2}-\frac{3719}{3360}hf_{n+3}-\frac{9159}{8960}hf_{n+4}-\frac{1161}{2240}hf_{n+5}+\frac{5619}{89600}hf_{n+6}-\frac{13}{640}h^2y''_{n+6}$$
$$y_{n+3}=y_{n+5}+\frac{1}{4536}hf_n-\frac{1}{315}hf_{n+1}+\frac{131}{5040}hf_{n+2}-\frac{1171}{2835}hf_{n+3}-\frac{3067}{2520}hf_{n+4}-\frac{134}{315}hf_{n+5}+\frac{491}{15120}hf_{n+6}-\frac{1}{108}h^2y''_{n+6}$$
$$y_{n+4}=y_{n+5}-\frac{53}{48384}hf_n+\frac{341}{33600}hf_{n+1}-\frac{2393}{53760}hf_{n+2}+\frac{4033}{30240}hf_{n+3}-\frac{16789}{26880}hf_{n+4}-\frac{3611}{6720}hf_{n+5}+\frac{154913}{2419200}hf_{n+6}-\frac{13}{640}h^2y''_{n+5}$$
$$y_{n+6}=y_{n+5}-\frac{731}{725760}hf_n+\frac{179}{20160}hf_{n+1}-\frac{5771}{161280}hf_{n+2}-\frac{8131}{90720}hf_{n+3}-\frac{13823}{80640}hf_{n+4}+\frac{12079}{20160}hf_{n+5}+\frac{247021}{483840}hf_{n+6}-\frac{275}{3456}h^2y''_{n+6} \tag{13}$$

3.3.7 Seven- Step Blended Block Linear Multistep Methods (BBLMM)

$$y_n=y_{n+6}-\frac{199}{700}hf_n-\frac{1413}{875}hf_{n+1}+\frac{27}{350}hf_{n+2}-\frac{89}{35}hf_{n+3}+\frac{99}{140}hf_{n+4}-\frac{459}{175}hf_{n+5}+\frac{401}{1750}hf_{n+6}+\frac{9}{175}hf_{n+7}-\frac{9}{25}h^2y''_{n+7}$$
$$y_{n+1}=y_{n+6}+\frac{1625}{217728}hf_n-\frac{25685}{72576}hf_{n+1}-\frac{250}{189}hf_{n+2}-\frac{141875}{217728}hf_{n+3}-\frac{96875}{72576}hf_{n+4}-\frac{17125}{24192}hf_{n+5}-\frac{5665}{9072}hf_{n+6}-\frac{125}{10368}hf_{n+7}+\frac{425}{2592}h^2y''_{n+7}$$
$$y_{n+2}=y_{n+6}-\frac{52}{42525}hf_n+\frac{1328}{70875}hf_{n+1}-\frac{1892}{4725}hf_{n+2}-\frac{10288}{8505}hf_{n+3}-\frac{2356}{2835}hf_{n+4}-\frac{5392}{4725}hf_{n+5}-\frac{30928}{70875}hf_{n+6}+\frac{16}{14175}hf_{n+7}+\frac{104}{2025}h^2y''_{n+7}$$
$$y_{n+3}=y_{n+6}+\frac{11}{22400}hf_n-\frac{657}{112000}hf_{n+1}+\frac{27}{700}hf_{n+2}-\frac{2021}{4480}hf_{n+3}-\frac{5031}{4480}hf_{n+4}-\frac{20871}{22400}hf_{n+5}-\frac{7321}{14000}hf_{n+6}-\frac{99}{22400}hf_{n+7}+\frac{81}{800}h^2y''_{n+7}$$
$$y_{n+4}=y_{n+6}-\frac{29}{170100}hf_n+\frac{139}{70875}hf_{n+1}-\frac{107}{9450}hf_{n+2}+\frac{421}{8505}hf_{n+3}-\frac{5237}{11340}hf_{n+4}-\frac{5321}{4725}hf_{n+5}-\frac{64003}{141750}hf_{n+6}-\frac{1}{2025}hf_{n+7}+\frac{127}{2025}h^2y''_{n+7}$$
$$y_{n+5}=y_{n+6}+\frac{1201}{5443200}hf_n-\frac{20609}{9072000}hf_{n+1}+\frac{52}{4725}hf_{n+2}-\frac{37631}{1088640}hf_{n+3}+\frac{32233}{362880}hf_{n+4}-\frac{302429}{604800}hf_{n+5}-\frac{633277}{1134000}hf_{n+6}-\frac{8563}{1814400}hf_{n+7}+\frac{7297}{64800}h^2y''_{n+7}$$
$$y_{n+7}=y_{n+6}-\frac{6031}{5443200}hf_n+\frac{99359}{9072000}hf_{n+1}-\frac{943}{18900}hf_{n+2}-\frac{153761}{1088640}hf_{n+3}-\frac{105943}{362880}hf_{n+4}+\frac{341699}{604800}hf_{n+5}+\frac{449527}{1134000}hf_{n+6}+\frac{416173}{1814400}hf_{n+7}+\frac{33953}{64800}h^2y''_{n+7} \tag{14}$$

2.1 Stability analysis of the methods (see[14])

In the spirit of [18], the local truncation error associated with the block method is the linear difference operator

$$L[Y(x):h] = \sum_{j=0}^{k}\{\alpha_j Y(x+jh) - hY'\beta_j - h^2 hY'Y''\gamma_j(x+jh)\} \tag{15}$$

We assume that $Y(x)$ is sufficiently differentiable, and so the terms of (5) can be expanded by Taylor series about x to give the expression

$$L[Y(x):h] = C_0 hZ(x) + C_1 Z'(x) + \cdots + C_q Z^q(x) + \cdots \tag{16}$$

Where $C_0 = \sum_{j=0}^{k}\alpha_j$

$$C_1 = \sum_{j=0}^{k} j\alpha_j - \sum_{j=0}^{k}\beta_j$$

$$C_2 = \frac{1}{2}\sum_{j=0}^{k} j^2\alpha_j - \sum_{j=0}^{k} j\beta_j - \sum_{j=0}^{k}\gamma_j$$

$$C_q = \frac{1}{q}\sum_{j=0}^{k} j^q\alpha_j - \frac{1}{(q-1)!}\sum_{j=0}^{k} j^{q-1}\beta_j - \frac{1}{(q-2)!}\sum_{j=0}^{k} j^{q-2}\beta_j \qquad q = 5,6,.. \tag{17}$$

A block method is said to be of order p if $\overline{C_0} = \overline{C_1} = \overline{C_2} = \cdots = \overline{C_p} = C_{p+1} = 0, \overline{C_{p+2}} \neq 0$
$\overline{C_{p+2}}$ is the error constant and local truncation error of the methods

The fifth, sixth and seventh step method has order

$$p = (6,6,6,6,6)^T$$
$$p = (7,7,7,7,7,7)^T$$
$$and \;\; p = (8,8,8,8,8,8,8)^T$$

and error constants of $C_8 = \left(\frac{743}{100000}, \frac{-432}{10000}, \frac{354}{1000000}, \frac{-574}{100000}, \frac{863}{1000000}\right)^T$

$$C_9 = \left(\frac{702}{100000}, \frac{-436}{100000}, \frac{731}{1000000}, \frac{375}{100000}, \frac{528}{1000000}, \frac{59}{10000}\right)^T$$

$$C_{10} = \left(\frac{1}{200}, \frac{813}{1000000}, \frac{-204}{1000000}, \frac{145}{100000}, \frac{-461}{10000000}, \frac{1}{10000}, \frac{615}{1000000}\right)^T$$

Respectively. Since the block methods are all of order $P \geq 1$, they are all consistent. [19]

3 Zero-stability of the Block Methods

Following the work of [14], we observed that the seven-step block method is zero stable as the roots of the equation $det(r(A - Cz - D1z^2) - B) = 0$ are less than or equal to 1.

Since the block method is consistent and zero-stable, the method is convergent [19].

4 Region of Absolute Stability

Solving the characteristic equation $det(r(A - Cz - D/z^2) - B)$ for r, we obtain the stability functions for k=7, 6, and k=5 respectively:

$$R(z) = \left(\begin{array}{l} (293549(3257z^7 + 96865z^6 - 7515311z^5 - 10553225z^4 \\ + 435857021z^3 + 45950311821z^2 + 2190780031)) / \\ (25962811280z^9 - 826829700z^8 - 2760898945z^7 + \\ 851407818z^6 + 509499019503z^5 - 83077823942880z^4 \\ - 248624617895462z^3 - 4006457142890z^2 + \\ 75052181186900z + 573907215521) \end{array} \right) \tag{18}$$

$$R(z) = \left(\begin{array}{l} (181440(3348z^6 + 73656z^5 - 8015677z^4 - 107373258z^3 \\ + 585043713z^2 + 8709120)) / (118296521280z^8 - 398268079200z^7 \\ - 12036740989815z^6 + 17140481870582z^5 + 203807488019508z^4 \\ - 83077823942880z^3 - 286454982461760z^2 + 152180571916800z \\ + 1580182732800) \end{array} \right) \tag{19}$$

$$R(z) = \left(\begin{array}{l} (255230114(25410z^5 - 73585677z^4 \\ + 252291758z^3 - 653043342z^2 + 90021753)) \\ /(333699531580z^6 + 17140481870582z^5 + \\ 29733481123z^4 - 51965523492880z^3 - \\ 7604546164382z^2 + 21684689211z + 1580182732800) \end{array} \right) \tag{20}$$

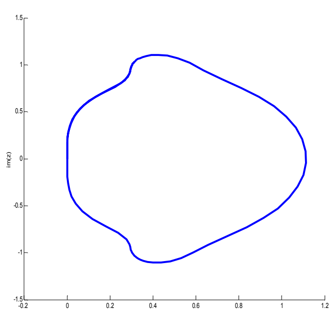

Fig.1a

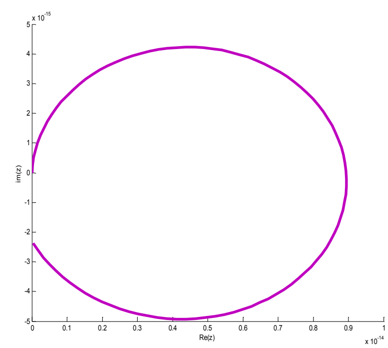

Fig. 1b

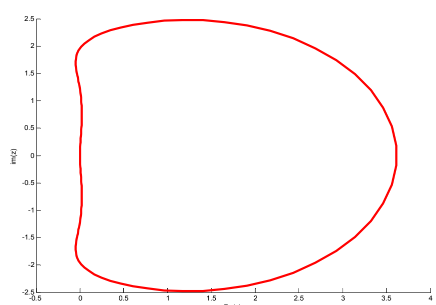

Fig. 1c

Fig. 1a is the Region of Absolute Stability of the Blended Block Methods for K=7, Fig. 1b is the Region of Absolute Stability of the Blended Block Methods for K=6, and Fig. 1c is the Region of Absolute Stability of the Blended Block Methods for K= 5. These absolute stability regions are all A- Stable since they contain the left-hand half complex plain. The stability region is all the regions outside the shape. The circle or oval shape is the excluded region for the stability region.

4.1 Numerical experiment

In this section, we shall test the new A-stable block methods for k=5,6 and 7 to ascertain their suitability for solving (1)

Problem 1 Irregular Heartbeat and Lidocaine Model

The irregular heartbeat and Lidocaine model is expressed mathematically by the following $ivp\ y_1' = -0.09\,y_1 + 0.038\,y_2$ and $y_2' = 0.066\,y_1 - 0.038\,y_2, y_1(0) = y_2(0) = y_0,\ y_0 =$ Maximum Safe Dosage $=$ 3mw/kg^3 $0 \le x \le 700,\quad h = 0.1$

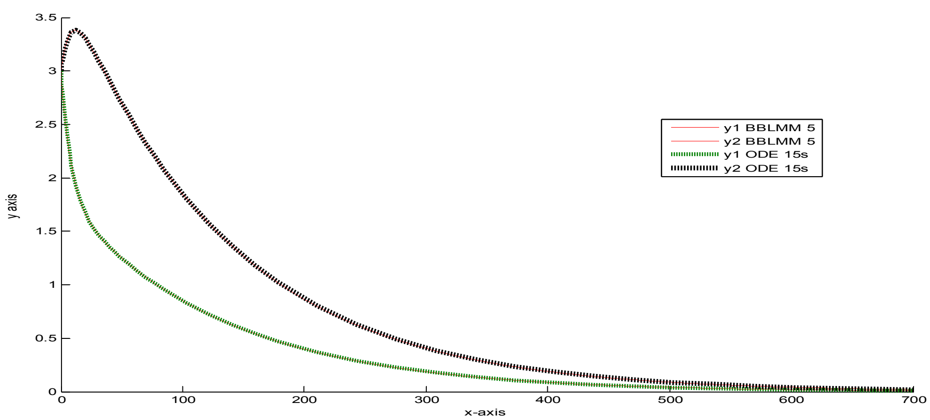

Fig. 2a

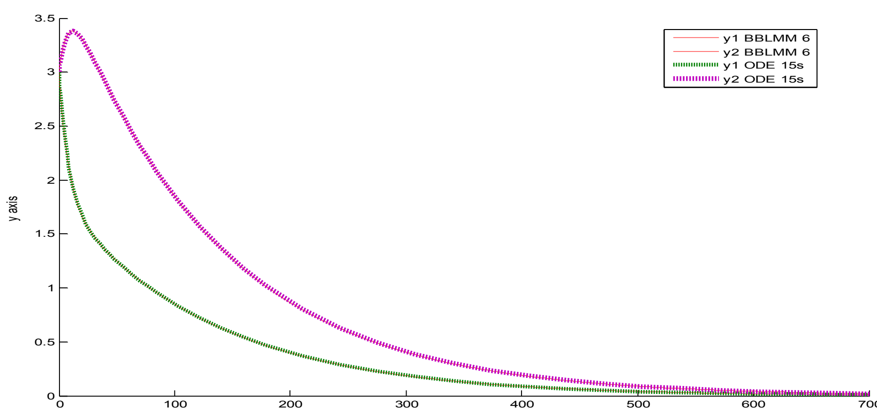

Fig. 2b

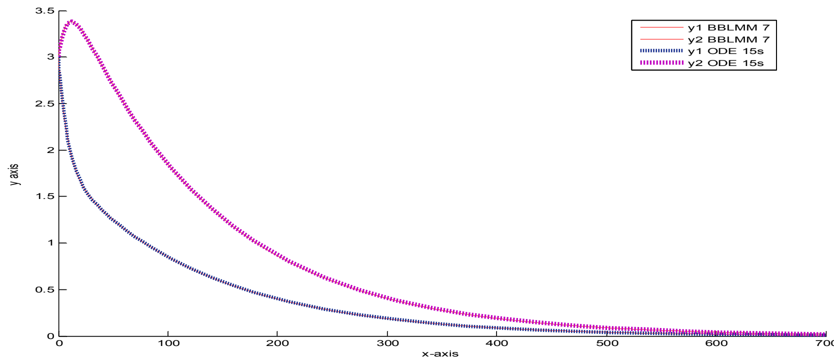

Fig. 2C

Problem 2 Stiff Linear System

$y_1' = -500000.5\, y_1 + 4.99999.5\, y_2$
$y_2' = 499999.5 y_1 - 500000.5\, y_2$
$y_1(0) = 0, y_2(0) = 2 \quad 0 \le x \le 100, \quad h = 0.1$

Theoretical solution is given by

$y_1(t) = -e^{t\lambda_1} \pm e^{t\lambda_2},\ y_2(t) = e^{t\lambda_1} + -e^{t\lambda_2} s$

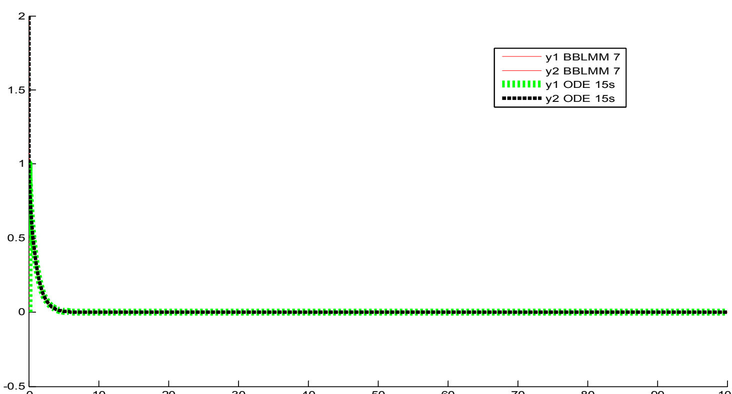

Fig. 3a

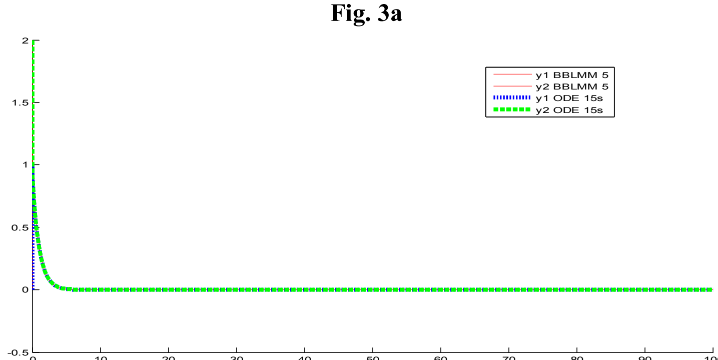

Fig. 3b

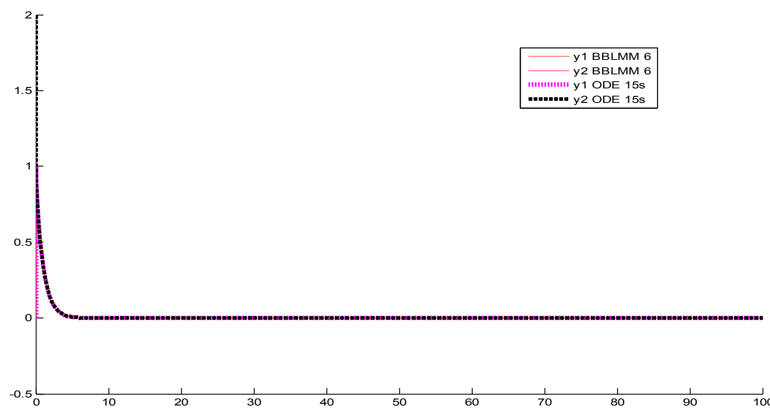

Fig. 3c

Fig. 3a, Fig 3b and Fig 3c are the solution curves for problem 1 using blended blocks for k=5,6 7 respectively

Table 1. Absolute Errors of the First Component for Problem 2 using BBLMMS

h=0.1	**BBLMMS 5**	**BBLMMS 6**	**BBLMM 7**
x	**Error**	**Error**	**Error**
0	0.0000E+00	0.0000E+00	0.0000E+00
0.1	4.0593E-06	1.7934E-06	3.2012E-06
0.2	3.5214E-06	2.2618E-07	5.3954E-08
0.3	6.1853E-08	4.3728E-08	2.8396E-08
0.4	3.4290E-09	3.2684E-09	2.04333E-10
0.5	5.6238E-09	2.2573E-09	1.8243E-10
0.6	1.53128E-10	6.1744E-10	4.228E-12
0.7	1.1353E-11	8.5006E-11	3.0913E-12
0.8	1.54097E-11	5.2304E-11	1.1827E-12
0.9	9.8532E-12	4.8429E-14	2.8920E-14
1.0	4.3851E-14	1.0641E-16	4.38111E-16
1.1	1.4327-16	5.5525E-18	2.4820E-18
1.2	2.7648E-16	9.4352E-18	3.5266E-18

5 Conclusion

The New Class of blended block second derivative linear multistep methods has been constructed through the multistep collocation approach for the solution of stiff systems. The analysis of the stability properties shows that the methods are all A-stable and convergent. Numerical experiments reveal the efficiency and accuracy of the newly constructed methods in solving Problem 5.4 which is a model of the relationship between Lidocaine and Irregular Heartbeat. Lidocaine belong to a group of drugs known as anti-arrhythmic which work by preventing sodium from being pumped out on the cells of the Heart to help the Heartbeat normally.

From our solution curves, it was observed that normalcy in the Heartbeat can be attained with the use of Lidocaine within the correct dosage. Our solution curves coincide with the solutions of ODE 15s. From the solution curve, there are four lines in the legend while the graph is showing two lines signifying that the solution curve for ODE 15 and that of our methods are on each other.

The Solutions curves are In terms of $y_{1\ and\ y_2}$ please.

Tables 1 show that our methods performed well with marginal absolute error constants.

Generally BBLMMs for step numbers k=5,6 and 7 approximate well the solutions of the linear and non-linear stiff system of Ordinary Differential Equations as compared with the well-known ODE solvers ODE 15s solution curves on the same axes and the exact solutions in problem 2 from the table of absolute error values.

Competing Interests

Authors have declared that no competing interests exist.

References

[1]. Gear CW. Hybrid methods for initial value problems in ordinary differential equations. SIAM J. Numer. Anal. Ser B, 2. 1965;69–86.

[2]. Lambert JD. Computational methods in ordinary differential equations. John Willey. New York; 1973.

[3]. Butcher JC. On the convergence of numerical solutions to ordinary differential equations. Math. Comp. 1966;20:1-10.

[4]. Enright WH. Second derivative multistep methods for stiff ordinary differential equation. SIAM J. Numer. Anal. 1974;11:321-331.

[5]. Odekunle MR, Adesanya AO, Sunday J. A 4-point block method for direct integration of first order ordinary differential equations. International Journal of Engineering Research and Applications. 2012;2(5):1182-1187.

[6]. Ezzeddine AK, Hojjati G. Third derivative multistep methods for stiff systems. International Journal of Nonlinear Science. 2012;14:443-450.

[7]. Brugnano L, Trigiante D. Solving differential problems by multistep initial and boundary value methods. Gordon and Breach Science Publishers, Amsterdam; 1998.

[8]. Chollom JP, Kumleng GM, Longwap S. High order block implicit multi-step (HOBIM) methods for the solutions of stiff ordinary differential equations. International Journal of Pure and Applied Mathematics. 2014;96(4):483-505.

[9]. Kumleng GM, Sirisena UW, Chollon JP. A class of a-stable order four and six linear multistep methods for stiff initial value problems. Mathematical Theory and Modeling. 2012;3(11):94-102.

[10]. Genin Y. An algebraic approach to A-stable linear multistep-multi derivative integration formulas. BIT. 1974;14:382-406.

[11]. Gamal AF, Iman HI. New efficient second derivative multistep methods for stiff systems. Applied Mathematical Modelling. 1998;23:279-288.

[12]. Sahi RK, Jator SN, Khan NA. A Simpson's type second derivative method for stiff systems. International Journal of Pure and Applied Mathematics. 2012;81(4):619-633.

[13]. Mehdizadeh KM, Nasehi ON, Hojjati G. A class of second derivative multistep methods for stiff systems. Acta Universitatis Apulensis. 2012;30:171-188.

[14]. Ehigie JO, Okunuga SA. L(α)-stable second derivative block multistep formula for stiff initial value problems. International Journal of Applied Mathematics. 2014;44(3):07.

[15]. Dahlquist G. A special stability problem for linear multistep methods. 1963;(Bit 3):27-43.

[16]. Lie I, Norsett. Super convergence for multistep collocation. Mathematics of Computation. 1989;52(185):65-79.

[17]. Onumanyi P, Awoyemi DO, Jator SN, Sirisena UW. New linear multistep methods with continuous coefficients for first order initial value problems. Journal of the Nigerian Mathematical Society. 1994;13:37–51.

[18]. Fatunla SO. Block method for second differential equations. International Journals of Computer Mathematics. 1991;41(1):55-66.

[19]. Henrici P. Discrete variable method in ordinary differential equations. John Willey. New York; 1962.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciencedomain.org/review-history/25323