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A Markov Chain Model for Wet and Dry Spell Probabilities at Yola, Adamawa State.

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Abstract

The theoretical probabilities of wet and dry spells were derived from Markov Chain Model using the threshold level of 0.25mm per day for a period of 20 years to predict the length of dry spell and wet spell during the rainy season (April to September) at Gyawana meteorological station in Yola, North Eastern Nigeria. The equilibrium probabilities for the station over the 20- year period are $\pi = (0.76, 0.24)$.This implies that the probability of dry day occurrence regardless of the weather conditions of the previous days is 0.76. The mean dry and wet duration for the station were found to be 5.52 and 1.72 days respectively. The mean weather cycle was 7.44. This information can be used to select the best planting date by avoiding the period of high risk of long dry period near the beginning of the rainy season always experienced in northern Nigeria.

Keyword: Dry spell, Wet spell, Planting, Markov chain model.

Introduction

Rainfall in Yola is characterized by a high degree of variability and it is the element of climate most influential in determining the variety and abundance of flora and fauna, land use, economic development and practically all aspects of human activity. Hence a comprehensive analysis of rainfall data is a crucial component in water management, irrigation management and agricultural production.

In recent years there has been considerable success in using the Markovian model to fit statistical distribution to meteorological observations. Most of these papers have dealt exclusively with occurrence of wet and dry days. Studies employing discrete time and stationary transition structure include those by Katrina (2002), Sayang and Abdul (2009), Ochola and Kerkides (2003) and Annick (1987). Schoof and Pryor (2008) considered the identification of model order using the Bayesian information criteria. Dahale *et al.*, (1994) considered persistence in rainfall occurrence, but only recently have the Markovian models been applied to climatological processes exhibiting sequential pattern in Nigeria.

The Markov chain properties possessed by successive meteorological events can aid in the future prediction based on the present. The purpose of this paper is to obtain various states of daily precipitation occurrence in Gyawana station in Yola, north eastern Nigeria, using the two states, first order Markov chain model, which will help farmers to select the best planting date in order to avoid the period of high risk of long dry spell near the beginning of the rainy season experienced in this part of the country. This research therefore becomes necessary and important because the application of this technique of rainfall analysis has not been constantly practiced in this part of the globe.

Materials and Method

The Study Area

Gyawana is located near Numan, Adamawa State, Nigeria, on latitude $9^{0}31^{1}$N and longitude $11^{0}49^{1}$E. The climate is characterized by two distinct seasons: wet (April to October) and dry (November to March). The mean annual rainfall is 905 mm, the wettest months being August and September (Binbol *et al.*, 2006). The beginning and end of the rainy season is marked by highly inconsistent linesqualls and thunderstorm activities (Binbol, 2005). This coupled with high temperatures throughout most of the year (mean temperature,26.9^{0}C; range, 18^{0}C- 40^{0}C. result in high evaporative tendency and prolong dry spells which invariably affects planting in the area. Rainfed agriculture is the major occupation of people in the study area.

Data

Daily records of rainfall for 20 years (1981-2000) were transcripted from the Meteorological station in Gyawana, Numan, north eastern Nigeria. The data include a series of zeros which can be interpreted as corresponding to a period of dry days just before the commencement of the rainy seasons. Days were classified as wet (W) or dry (D) according to whether there had or had not been at least 0.25mm of precipitation recorded in a day.

The data for the purpose of this study were tested for homogeneity. When considering rainfall series over a long period of time, one must be aware that the data collected may not reflect uniform condition. Therefore, before performing different analysis the homogeneity of observed data with respect to non-climatic influences must therefore be assessed.

In this paper, the cumulative sum techniques (Hayes *et al.*, 2011) were used for the detection and quantification of jumps to determine homogeneity in the observed rainfall data. This is achieved by plotting the partial sum of the departures from the mean (See figure 1). Using the partial sum together with the history of the station, the long term annual rainfall data were split into two sub-series, as follows 1981 - 1991 and 1992 - 2000. The main decisive factor for splitting into two sub-series is that each sub-series have a different pattern of rainfall. Estimates of the annual means μ_1 and μ_2 sub-series are 943.958 mm and 891.433 mm respectively.

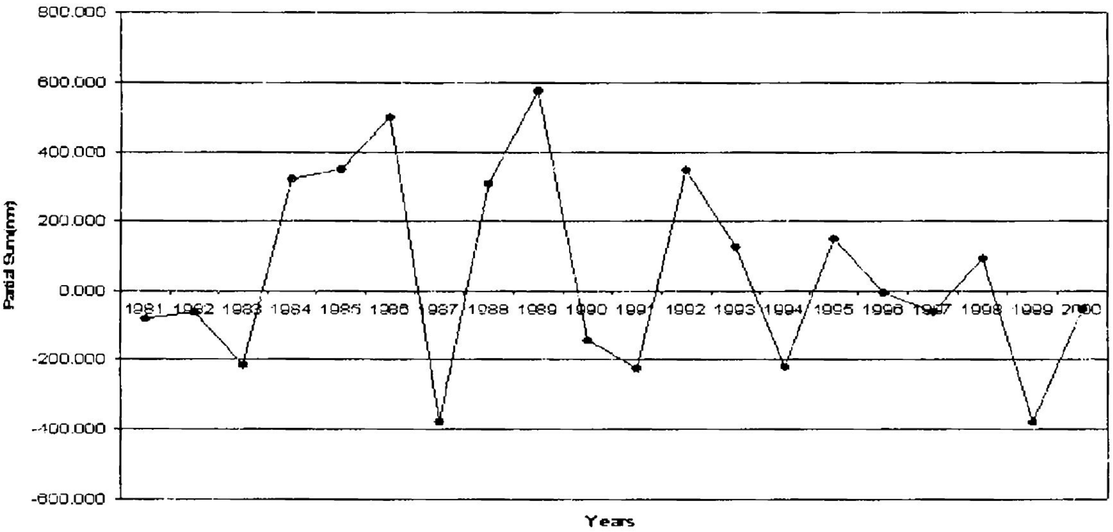

Figure 1: Partial Sum of Departures from the Mean of Annual Totals (n=20)

Homogeneity was tested under the assumptions of equal variances and normality of the annual rainfall totals using Snedecor's F- statistics.

$H_0 : \mu_1 = \mu_2$

$H_1 : N_1 \neq N_2$

Where H_0 is the null hypothesis and H_1 is the alternative hypothesis

The result of the calculated F-value (critical value) is 0.41940 and the table value at 0.05 significance level is 3.14. Hence we accept the null hypothesis and conclude that the station is homogeneous.

Data Analysis

In this paper, we consider a two-state Markov chain, with the following transition probabilities. (i.e Dry and Wet state).

$$P = P_{ij} = \begin{array}{c} \\ Dry \\ Wet \end{array} \begin{array}{c} \begin{array}{cc} Dry & Wet \end{array} \\ \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \end{array} \qquad (1)$$

The tally matrix n_{ij} of the frequencies of transitions between the two successive states is given by

$$N = n_{ij} = \begin{array}{c} \\ Dry \\ Wet \end{array} \begin{array}{c} \begin{array}{cc} Dry & Wet \end{array} \\ \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \end{array} \qquad (2)$$

n_{ij} represent the number of transitions from one state to another while P_{ij} represent the probabilities from one state to another.

Application of the Model for Daily Precipitation Occurrences

Our approach followed that of Ochola and Kerkides (2003) and Korve *et al.,* (2006). The n-step transition probability P_{ij}^{n} is defined as the probability that a process in state *i* will be in state *j* after n additional transitions. That is

$$p_{ij}^{n} = p(x_{n+m} = j | x_m = i) \quad n \geq 0, \quad i, j \geq 0 \qquad (3)$$

The n-step transition matrix may be obtained by multiplying the matrix P by itself n times. If $\pi = \pi P$ after n-steps, then the distribution is stationary and $\pi = (\pi_0, \pi_1)$ is called the equilibrium probability. This implies that the state occupation probabilities are independent of the initial conditions. In the case of a two-state first order Markov chain model for daily precipitation occurrence, the equilibrium probability means that the wet day and dry day occupation probabilities are independent of the previous weather conditions. According to Korve *et al* (2006), if π_0 is the wet day probability component and π_1 is the dry day probability component, then

$$_0 = \frac{p_{21}}{p_{21} + p_{12}}, \qquad _1 = \frac{p_{12}}{p_{21} + p_{12}} \qquad (4)$$

Given any state *i*, we let g_i denote the probability that starting from state *i*, the process re-enter state *i*. State *i* is said to be transient if $g_i < 1$ and recurrent if $g_i = 1$. If state *i* is transient, then the probability that the process will never again enter state *i* is $(1 = g_i)$. Also starting from state *i*, the probability that the process will be in state *i* for exactly *i* time *n* time period is given by $F_i^{(n-1)}(1 - g_i)$ $n \geq 1$. Hence the probabilities that the process will maintain *n* dry (D) days and *n* wet (W) days are given by

$$P(D = n) = p_{11}^{(n-1)}(1 - p_{11}) \; : \; P(W = n) = p_{22}^{(n-1)}(1 - p_{22}) \qquad (5)$$

The mean time period for dry (D) days and wet (W) days are

$$E(D) = \frac{1}{1 - p_{11}} \qquad E(W) = \frac{1}{1 - p_{22}} \qquad (6)$$

The weather cycle C is given by

$$E(C) = E(W) + E(D) \qquad (7)$$

The probability of wet days among any n days is given by

$$P(W|n) = P(s|n, W) + (1 - p)P(s|n, D) \qquad (8)$$

Where $P(s|n, W)$ is the probability of the exact number of wet days among n days following a wet day and $P(s|n, D)$ following a dry day.

Results

The result of the analysis between wet and dry day precipitation are shown

Table 1: Tally matrix and Transition Probability of the Occurrence of wet and dry days following the same conditions on the previous days for twenty (20) years at Gyawana Meteorological station of Yola, North Eastern Nigeria.

Period	Tally Matrix $\left(N = n_{ij}\right)$	Transition Probability $\left(P = p_{ij}\right)$
1981 - 2000 (20 years)	$\begin{bmatrix} 5713 & 1217 \\ 1217 & 865 \end{bmatrix}$	$\begin{bmatrix} 0.82 & 0.18 \\ 0.58 & 0.42 \end{bmatrix}$
1981 - 1991 (Series 1)	$\begin{bmatrix} 2992 & 662 \\ 662 & 383 \end{bmatrix}$	$\begin{bmatrix} 0.82 & 0.18 \\ 0.63 & 0.37 \end{bmatrix}$
1992 - 2000 (Series 2)	$\begin{bmatrix} 2721 & 555 \\ 555 & 482 \end{bmatrix}$	$\begin{bmatrix} 0.83 & 0.17 \\ 0.54 & 0.46 \end{bmatrix}$

We compute the equilibrium probabilities using equation (4). This gives

$$\pi_0 = \frac{p_{21}}{p_{21}+p_{12}} = \frac{0.58}{0.58+0.18} = 0.7632$$

$$\pi_1 = \frac{p_{12}}{p_{21}+p_{12}} \quad \frac{0.18}{0.58+0.18} = 0.2368$$

We compute the mean time period for dry (D) days and wet (W) days for the station from equation (6).

$$E(D) = \frac{1}{1-p_{11}} = \frac{1}{1-0.82} = 5.556 \cong 6\text{days}$$

$$E(W) = \frac{1}{1-p_{22}} = \frac{1}{1-0.42} = 1.7241 \cong 2\text{days}$$

We compute the weather cycle C from equation (7) as follows

$$E(C) = 5.5556 + 1.7241 = 7.2797 \cong 7 \ days$$

A summary of the above result for the two sub-series is shown in table 2 below.

Table 2: Equilibrium Probabilities and Weather Cycle for sub-series.

Period	π_0	π_1	*E(D)*	*E(W)*	*E(C) = E(W)+E(D)*
1981 2000	0.76	0.24	5.56	1.72	7.28 (≅ 7 days)
1981 1991 (Series 1)	0.67	0.33	5.56	1.59	7.15 (≅ 7 days)
1992 2000 (Series 2)	0.64	0.36	5.88	1.85	7.73 (≅ 8 days)

We now compute the probabilities that the process will maintain dry (D) days and wet (W) days for the given period (1981 - 2000), using equation (8).(See Table 3 and Figure2)

Table 3: Probabilities of Dry and Wet Durations.

State	1	2	3	4	5	6	7	8
Dry	0.180	0.148	0.121	0.099	0.081	0.067	0.055	0.045
Wet	0.580	0.244	0.102	0.043	0.018	0.008	0.003	0.001

State	9	10	11	12	13	14	15
Dry	0.037	0.030	0.025	0.020	0.017	0.014	0.015
Wet	0.001	0.0	0.0	0.0	0.0	0.0	0.0

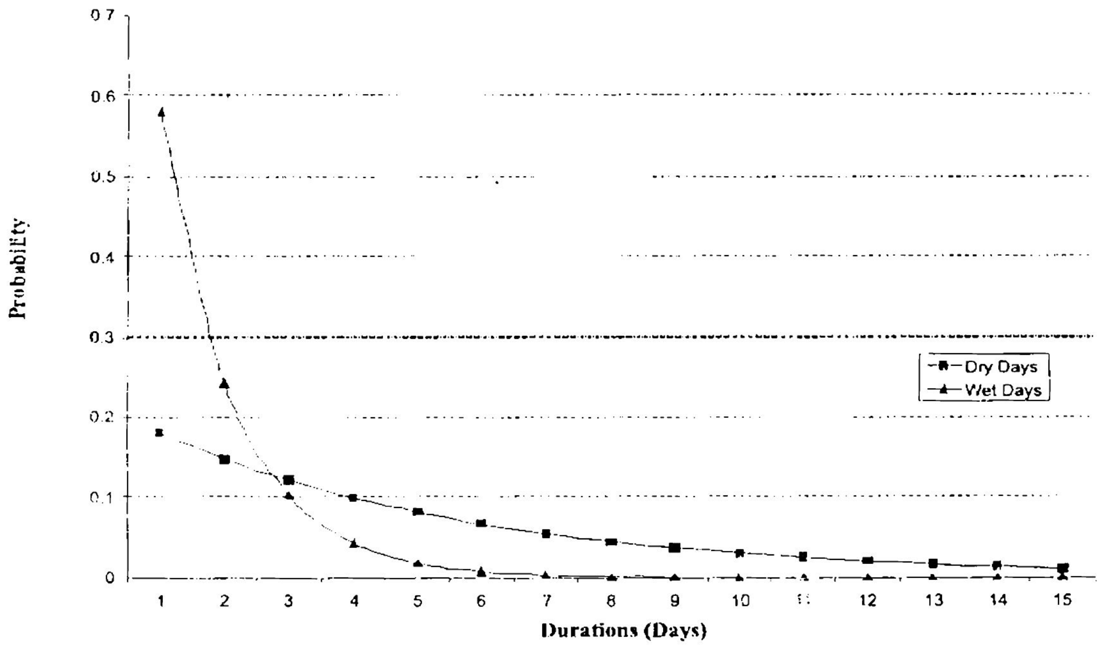

Figure 2: Probabilities for Dry Days and Wet Days Durations.

Discussion

Table 1 shows that there are 9,012 total transitions for the station during the 20 years period. The probability of transition from dry day to dry day is 0.82 and from dry day to wet day is 0.18, the former being remarkably larger than the latter. The probability of transition from wet day to wet day is 0.42 and from wet day to dry day is 0.58, hence the latter is greater than the former.

Table 2 shows the mean dry day duration and wet day duration for the station during the period under study. It is clear even from the sub-series that the mean wet day duration is less than two days and the mean dry day duration is approximately 6 days in Yola. The calculation of weather cycle for Yola indicates a cycle of approximately 7 days. Apparently from Table 2, the dry day probability elements (π_0) are on the average about 3 times larger than the wet day probability element (π_1) when we consider the entire series as a whole.

Table 3 shows the probabilities that maintained dry or wet days for n days period. Distribution of daily rainfall for Yola shows that the dry day duration appears up to about two weeks (12 days) with probability 0.008 (about 0.01) (See Figure 2). The mean dry day duration was found to be between 5 to 6 days. On the average wet day duration would fall within the range of 2 to 4 days.

Conclusion

The rainfall properties for Yola has been calculated by the method of Markov chains using the threshold level of 0.25mm per day for a period of 20 years to predict the length of dry spell and wet spell during the rainy season (April to September). The following results emerged

- Statistical test confirmed that the data collected are homogeneous and hence reflect uniform conditions for the analysis carried out in this paper.
- The equilibria probabilities for the station over the 20 years period are $\pi = (0.76, 0.24)$. This implies that the probability of dry day occurrence regardless of the weather conditions of the previous days is 0.76
- The mean dry and wet duration for the station are 5.56 and 1.72 days respectively. The mean weather cycle is 7.28.

Strategic management is therefore needed by analyzing the time series, start of rainy season and water balance of the long historical data for the prediction of a season to be wet or dry in any locality. Techniques like Markov chain can be very useful for estimation of the material cost of risk of long dry spells usually experienced in tropical areas. The threshold values separating wet days from dry days can also be estimated in future research using Markov chain models.

References

Annick, D. (1987). The variations of dry spells in Marseilles from 1865 to 1984. *International Journal of Climatology* 7:6, 541-551

Binbol, N.L. (2005). Seasonal Variation of Linesqualls and Thurderstorm activities in Yola. *Journal of Natural and Applied Sciences,* 1 (1): 59-65.

Binbol, N.L., Adebayo, A.A. and Kwon Ndong, E.H. (2006). Influence of Climatic factors on the growth and yield of Sugar cane at Numan, Nigeria; *Climatic Research,* 32: 247-252.

Dahale, S.D., Panchawagh, N., Singh, S. V., Ranatunge, E. R. and Brikshavana, M. (1994). Persistence in rainfall occurrence over Tropical south-east Asia and equatorial Pacific. *Theoretical and Applied Climatology*, 49:1, 27-39.

Hayes, G.M., Ramirez, J. and Langley Hobbs, S.J. (2011). Use of the Cumulative Summation Technique to Quantitatively Assess a Surgical Learning Curve: Canine Total Hip Replacement. *Veterinary Surgery*, 40: 15-21.

Katrina, R. (2002). Black Spot and Rain: A Markov Chain Model for Appleby, New Zealand. New Zealand. *Geographer* 58:(2): 65-70.

Korve, K.N, Kassem, G.G., Imande, M.A. and Olanrewaju, S.O (2006). Markov Chain Properties of Daily Rainfall Occurrence in Northern Nigeria. *The Nigerian Journal of Tropical Geography.* 1(1): 27-34.

Ochola, W.O. and Kerkides, P. (2003). A Markov chain simulation model for predicting critical wet and dry spells in Kenya: analyzing rainfall events in the

Kano Plains. *Irrigation and Drainage*, 52: 327-342.

Sayang M.D. and Abdul, A.J. (2009). Fitting the distribution of dry and wet spells with alternative probability models. *Meteorology and Atmospheric Physics*, 104 (1-2): 13-27.

Schoof, J.T. and Pryor, S .C. (2008). On the Proper Order of Markov Chain Model for Daily Precipitation Occurrence in the Contiguous United States. *J. Appl. Meteor. Climatol.*, 47, 2477-2486.
