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OPTIMAL EQUIPMENT REPLACEMENT STRATEGIES WITH INDEX ONE ELECTRONIC IMPLEMENTATIONS: A CASE STUDY OF ZENITH PROCESSOR TIN MINING COMPANY, JOS, PLATEAU STATE, NIGERIA

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ABSTRACT

This article used backward dynamic programming recursions to investigate and obtain alternate optimal replacement policies with respect to the Machine fleet at Zenith Processor Tin Mining Company, Jos, Nigeria. The success of the investigation is largely attributed to the robust survey of the mining process, the deployment of analytic formulas for the determination of the set of feasible machine ages corresponding to each decision year to obviate the need for network diagrams and the exploitation of developed Excel solution templates to eliminate the associated cumbersome manual computations. These templates used the pertinent data, as well as the batch feasible ages at each stage as inputs to the solution process and are quite suitable for sensitivity analyses on starting machine ages with respect to mining systems, in particular and equipment replacement problems in general.

Keywords: Age transition diagrams; dynamic programming; decision period; decision symbols; EMS machine; equipment replacement optimization problems; mining system, pertinent data, set of feasible ages; Policy prescription; sensitivity analyses; Solution templates; stages.

1. BACKGROUND OF THE STUDY

The equipment replacement optimization (ERO) problem is an issue faced by virtually every industry and the problem has received much research attention (Gress, et al., [1]). Items which are under constant usage need replacement at appropriate times, due to diminishing efficiency occasioned by wear, tear and obsolescence with associated rising operating and maintenance (O&M) cost and decreasing salvage values. In the real-world, the equipment replacement problem involves the selection of two or more machines of one or more types from a set of several possible alternative machines with different capacities, cost of purchase and operation to produce efficiently.

Many planning and control problems involve a sequence of decisions that are made over time. The initial decision is followed by a second, the second by the third, and so on, and the process continues, perhaps indefinitely. According to Gress et al., [1], the original definition of dynamic programming was "planning over time" as the word dynamic describes situations that occur over time and programming a synonym for planning. The concern therefore is with decisions that relate to, and affect phenomena that are functions of time. A replacement policy is a policy of "Keep" or "Replace" actions, one for each period [1].

Although equipment replacement optimization problems have received much attention from earlier research works, the studies have focused on optimizing financial performance, that is, minimizing costs and maximizing profits, for the most part. In addition, earlier works in this field focused on the use of network diagrams for the determination of starting ages, which are prone to error and almost impossible for large horizon lengths. Moreover, even after the network diagrams have been furnished, the computations involved for the determination of the optimal replacement strategies and corresponding returns are quite prohibitive. There is therefore the need to develop analytic formulas for the set of feasible machine ages corresponding to each decision year to obviate the need for network diagrams. Furthermore, Excel solution templates need to be deployed to eliminate the associated cumbersome manual computations. These templates need to use the pertinent data, as well as the feasible ages at each stage as inputs to the solution process.

According to Vorster [2], surface mining equipment has a finite life and Mitchell [3] asserts that replacement theory seeks to answer the question: what is the optimum economic life of this piece of equipment? The dynamic programming deployed in this work used the idea of [3] and [2]. In this work, a comprehensive Dynamic Programming-based optimization methodology was developed to solve the equipment replacement optimization (ERO) problem on the replacement of machine at Zenith Processor Tin mining company, Jos. This work is intended to assess the economic value that a mining system may accrue due to machine replacement using the Equipment Replacement Dynamic Programming (ERDP) model as a decision making tool. This shall be achieved by: determining the most economic age of a machine in the mining system, calculating the maximum net income that is gained by the mining system through a ‘keep or Replace’ decision on machines within the sixteen-year period, ascertaining the optimal decision making policy of ‘Keeping or Replacing’ a machine system within sixteen-year period of its lifetime and designing a system of calculating approximate revenue in a mining system accrued from the replacement of machines.

The research efforts in this report will focus on and deploy an alternative solution platform in the form of ERO solution templates developed by Ukwu [4][5][6][7]. These templates provide optimal policy prescriptions and corresponding rewards for any given pertinent data such as the machine price, maintenance costs, revenues, salvage values. The templates appropriate the structure of the set of feasible machine ages corresponding to each decision year in [4].

2. THEORETICAL UNDERPINNING

Suppose that we are studying the machine replacement problem over a span of n years; at the start of each year, we decide whether to keep the machine or to replace it with a new one.

Let $c(t), r(t), s(t)$ represent the yearly revenue, operating cost and salvage value of a t-year old machine respectively. The cost of acquiring a new machine in any year is I.

2.1 Theoretical Analysis

In this section, the problem data, working definitions, elements of the DP model and the dynamic programming (DP) recursions are laid out as follows:

Equipment Starting age $= t_1$

Equipment Replacement age $= m$

S_i = The set of feasible equipment ages (states) in decision period i (say year i),

$i \in \{1, 2, \ldots, n\}$

$r(t)$ = annual revenue from a t – year old equipment

$c(t)$ = annual operating cost of a t – year old equipment

$s(t)$ = salvage value of a $t-$ year old equipment; $t = 0,1,\ldots,m$

I = fixed cost of acquiring a new equipment in any year

The elements of the DP are the following:

1. Stage i, represented by year $i, i \in \{1,2,\ldots,n\}$
2. The alternatives at stage (year) i. These call for keeping or replacing the equipment at the beginning of year i
3. The state at stage (year) i, represented by the age of the equipment at the beginning of year i. Let $f_i(t)$ be the maximum net income for years $i, i+1,\ldots,n-1, n$ given that the equipment is t years old at the beginning of year i.

Note: The definition of $f_i(t)$ starting from year i to year n implies that backward recursion will be used. Forward recursion would start from year 1 to year i.

The following theorem is applicable to the backward recursive procedure:

2.2 The recursive equation

The recursive equations are derived as

$$f_i(t) = \max \begin{cases} r(t) - c(t) + f_{i+1}(t+1), & \text{if KEEP} \\ r(0) + s(t) - I - c(0) + f_{i+1}(1), & \text{if REPLACE} \end{cases}$$

$$i \in \{1,2,\cdots,n-1\},\ f_{n+1}(x) = s(x),$$

where

x is the age of the machine at the beginning of stage $n+1$,

which coincides with the end stage n.

The theorem below details the computations of the feasible machine replacement ages corresponding to each decision year in an n-year planning horizon problem.

2.3 Theorem

Let S_i denote the set of feasible equipment ages at the start of the decision year i. Let t_1 denote the age of the machine at the start of the decision year i, that is, $S_1 = \{t_1\}$. If $m \le n$, and $t_1 \in \{0,1\}$, then for $i \in \{1,2,\ldots,n\}$,

$$S_i = \left\{ \min_{2 \le j \le i+t_1} \{j-1,\ m\} \right\}$$

This result is a corollary to Theorem 1, in [4]

3. MATERIALS, METHODS AND DISCUSSION

3.1 The Tin Mine Magnetic Separator System

The Tin Mine Shaft Plant that has been taken into consideration has seven conveyor belts, which are interconnected by various stations. All seven conveyor belts carry virtually the same weight, hence used for separation, just as the machine is used for separating minerals based on their magnetic properties, like iron, tin, sand, columbite etc. The magnetic separator separates minerals based on their magnetic properties.

Plate 1: The Conveyor Belt under consideration

3.2 Determination of the Input Parameters for the ERO Problem

The plant Engineer requires that a sixteen year old machine be replaced, the cost of a new machine (including replacement cost) is about ₦25,000,000. In deriving the revenue of an operational machine, we determined the revenue generated by the plant system per day since the downtime of a machine ultimately results in the loss of this same amount of revenue each day.

The mining system operates virtually every-day except Sundays for fifty-two weeks in a year, for 5-10 hours each day, making an average of ₦69450 per day for a six-day week period. This amounts to about ₦ 21,668,400 per annum. Empirical data suggest that this revenue decreases by about ten present every year due to ageing and the incorporating of risk factor. The operating cost of the (Electromagnetic separator) machine involves mainly the following: replacement of electrical or mechanical parts, replacement of bearings, replacement of cross over belts, and replacement of EMS machine which amount to about ₦1,500,000. The operating cost of a new machine (electromagnetic separator) is assumed to increase by about twenty percent each year due to increase in service requirements as the price of equipment gets older. The mine management assumes that they can approximately get about ₦17,000,000 from selling a one-year old electromagnetic separator machine, and this price may decrease by two percent as the EMS machine gets older each year. Table 1 shows the summary of data values for the revenue, operating cost and salvage value from a new ESM machine (0 year), up to a sixteen-year old EMS machine. I=25,000,000.

Table 1: Summary of Pertinent Data for the Revenue, Operating Cost and Salvage Value

Machine Age, *t*(Years)	**Operating Cost, *c*(*t*)**	**Revenue, *r*(*t*)**	**Salvage Value, *s*(*t*)**
0	1,500,000.00	21,668,400.00	
1	1,800,000.00	19,501,560.00	17,000,000.00
2	2,160,000.00	17,551,404.00	16,660,000.00
3	2,592,000.00	15,796,263.60	16,326,800.00
4	3,110,400.00	14,216,637.24	16,000,264.00
5	3,732,480.00	12,794,973.52	15,680,258.72
6	4,478,976.00	11,515,476.16	15,366,653.55
7	5,374,771.20	10,363,928.55	15,059,320.47
8	6,449,725.44	9,327,535.69	14,758,134.07
9	7,739,670.53	8,394,782.12	14,462,971.38
10	9,287,604.63	7,555,303.91	14,173,711.96
11	11,145,125.56	6,799,773.52	13,890,237.72
12	13,374,150.67	6,119,796.17	13,612,432.96
13	16,048,980.81	5,507,816.55	13,340,184.30
14	19,258,776.97	4,957,034.90	13,073,380.62
15	23,110,532.36	4,461,331.41	12,811,913.01
16	27,732,638.83	4,015,198.27	12,555,674.74

Note: There is no salvage value for a new EMS machine.

Solving the equipment replacement optimization (ERO) problem is equivalent to finding the longest route, that is the maximum revenue from the beginning of year one to the end of year sixteen.

Extensive code on Microsoft excel platform spread sheet has been developed by [6] and [7] to compute the optimal solutions and corresponding returns, using equations (1) and (2) at each stage. The outputs are shown in figures 5, 6, and 7, for the 0 starting age, and in figures 8 to 18 for selected non-zero starting ages. The age transition diagrams, with interpretations, as well as the optimal policy prescriptions and the corresponding maximum net profits are also furnished with respect to some selected starting machine ages.

Figure 1: Template Outputs for the Optimal Strategies and Rewards for Stages 16 to 9 for Starting Age 0

Equipment Replacement Problem Solution Temp					***n***	**Starting Age**	***n***		***c*(0)**	***r*(0)**	***s*(1)**	**Factor**					
Replacement Age =			16	**years**	16	0	16		1500000	21668400	17000000	0.000001					
Given Data (Millions)				**Stage**	16			Rates	1.2	0.9	0.98						
	***I* =**	25	***V(0) = r(0)-c(0) - I =***		-4.8316												
***t* (yrs.)**	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
***r*(*t*)**	21.6684	19.5016	17.5514	15.7963	14.2166	12.7950	11.5155	10.3639	9.3275	8.3947821	7.555304	6.79977	6.1198	5.5078	4.957	4.46133	4.015198266
***c*(*t*)**	1.5000	1.8000	2.1600	2.5920	3.1104	3.7325	4.4790	5.3748	6.4497	7.7396705	9.287605	11.1451	13.374	16.049	19.26	23.1105	27.73263883
***s*(t)**		17.0000	16.6600	16.3268	16.0003	15.6803	15.3667	15.0593	14.7581	14.462971	14.17371	13.8902	13.612	13.34	13.07	12.8119	12.55567474
K		34.3616	31.7182	29.2045	26.7865	24.4291	22.0958	19.7473	17.3408	14.8288236	12.157937	9.267081	6.08583	2.53222	-1.49	-6.09353	
R		29.1684	28.8284	28.4952	28.1687	27.8487	27.5351	27.2277	26.9265	26.6313714	26.342112	26.05864	25.7808	25.5086	25.242	24.98031	
***f*(t)**		34.3616	31.7182	29.2045	28.1687	27.8487	27.5351	27.2277	26.9265	26.6313714	26.342112	26.05864	25.7808	25.5086	25.242	24.98031	
D*		*K*	*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	
t		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
				Stage	15												
K		49.41976	44.595932	41.37293	38.954896	36.59754706	34.2642	31.9157	29.50918	26.9972236	24.326337	21.43548	18.2542	14.7006	10.679		
R		46.52996	46.18996	45.85676	45.530224	45.21021872	44.8966	44.5893	44.28809	43.9929314	43.703672	43.4202	43.1424	42.8701	42.603		
***f*(t)**		49.41976	46.18996	45.85676	45.530224	45.21021872	44.8966	44.5893	44.28809	43.9929314	43.703672	43.4202	43.1424	42.8701	42.603		
D*		*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*		
t		1	2	3	4	5	6	7	8	9	10	11	12	13	14		
				Stage	14												
K		63.89152	61.248164	58.73449	56.316456	53.95910706	51.6258	49.2773	46.87074	44.3587836	41.687897	38.79704	35.6158	32.0622			
R		61.58816	61.248164	60.91496	60.588428	60.26842272	59.9548	59.6475	59.3463	59.0511354	58.761876	58.4784	58.2006	57.9283			
***f*(t)**		63.89152	61.248164	60.91496	60.588428	60.26842272	59.9548	59.6475	59.3463	59.0511354	58.761876	58.4784	58.2006	57.9283			
D*		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*			
t		1	2	3	4	5	6	7	8	9	10	11	12	13			
				Stage	13												
K		78.94972	76.306368	73.79269	71.37466	69.01731106	66.684	64.3355	61.92895	59.4169876	56.746101	53.85524	50.674				
R		76.05992	75.71992	75.38672	75.060184	74.74017872	74.4266	74.1192	73.81805	73.5228914	73.233632	72.95016	72.6724				
***f*(t)**		78.94972	76.306368	75.38672	75.060184	74.74017872	74.4266	74.1192	73.81805	73.5228914	73.233632	72.95016	72.6724				
D*		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*				
t		1	2	3	4	5	6	7	8	9	10	11	12				
				Stage	12												
K		94.00793	90.778124	88.26445	85.846416	83.48906706	81.1557	78.8072	76.4007	73.8887436	71.217857	68.327					
R		91.11812	90.778124	90.44492	90.118388	89.79838272	89.4848	89.1774	88.87626	88.5810954	88.291836	88.00836					
***f*(t)**		94.00793	90.778124	90.44492	90.118388	89.79838272	89.4848	89.1774	88.87626	88.5810954	88.291836	88.00836					
D*		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*					
t		1	2	3	4	5	6	7	8	9	10	11					
				Stage	11												
K		108.4797	105.83633	103.3227	100.90462	98.54727106	96.2139	93.8654	91.45891	88.9469476	86.276061						
R		106.1763	105.83633	105.5031	105.17659	104.8565867	104.543	104.236	103.9345	103.639299	103.35004						
***f*(t)**		108.4797	105.83633	105.5031	105.17659	104.8565867	104.543	104.236	103.9345	103.639299	103.35004						
D*		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*						
t		1	2	3	4	5	6	7	8	9	10						
				Stage	10												
K		123.5379	120.89453	118.3809	115.96282	113.6054751	111.272	108.924	106.5171	104.005152							
R		120.6481	120.30808	119.9749	119.64835	119.3283427	119.015	118.707	118.4062	118.111055							
***f*(t)**		123.5379	120.89453	119.9749	119.64835	119.3283427	119.015	118.707	118.4062	118.111055							
D*		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*							
t		1	2	3	4	5	6	7	8	9							
				Stage	9												
K		138.5961	135.36629	132.8526	130.43458	128.0772311	125.744	123.395	120.9889								
R		135.7063	135.36629	135.0331	134.70655	134.3865467	134.073	133.766	133.4644								
***f*(t)**		138.5961	135.36629	135.0331	134.70655	134.3865467	134.073	133.766	133.4644								
D*		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*								
t		1	2	3	4	5	6	7	8								

Figure 2: Template Outputs for the Optimal Strategies and Rewards for Stages 8 to 1 for Starting Age 0

				Stage	8			
K		153.0678	150.42449	147.9108	145.49278	143.1354351	140.802	138.454
R		150.7645	150.42449	150.0913	149.76476	149.4447507	149.131	148.824
***f*(t)**		153.0678	150.42449	150.0913	149.76476	149.4447507	149.131	148.824
D*		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*
t		1	2	3	4	5	6	7
				Stage	7			
K		168.1261	165.4827	162.969	160.55099	158.1936391	155.86	
R		165.2362	164.89625	164.563	164.23651	163.9165067	163.603	
***f*(t)**		168.1261	165.4827	164.563	164.23651	163.9165067	163.603	
D*		*K*	*K*	*R*	*R*	*R*	*R*	
t		1	2	3	4	5	6	
				Stage	6			
K		183.1843	179.95445	177.4408	175.02274	172.6653951		
R		180.2945	179.95445	179.6213	179.29472	178.9747107		
***f*(t)**		183.1843	179.95445	179.6213	179.29472	178.9747107		
D*		*K*	*K/R*	*R*	*R*	*R*		
t		1	2	3	4	5		
				Stage	5			
K		197.656	195.01266	192.499	190.08095			
R		195.3527	195.01266	194.6795	194.35292			
***f*(t)**		197.656	195.01266	194.6795	194.35292			
D*		*K*	*K/R*	*R*	*R*			
t		1	2	3	4			
				Stage	4			
K		212.7142	210.07086	207.5572				
R		209.8244	209.48441	209.1512				
***f*(t)**		212.7142	210.07086	209.1512				
D*		*K*	*K*	*R*				
t		1	2	3				
				Stage	3			
K		227.7724	224.54262					
R		224.8826	224.54262					
***f*(t)**		227.7724	224.54262					
D*		*K*	*K/R*					
t		1	2					
				Stage	2			
K		242.2442						
R		239.9408						
***f*(t)**		242.2442						
D*		*K*						
t		1						
				Stage	1			
K	262.4126							
R	237.4126							
***f*(t)**	262.4126							
D*	*K*							
t	0							

3.2.1 Selected Age Transitions Diagrams for Optimal Policy Prescriptions and Corresponding Returns:

$$0K1K2K3R1K2K3R1K2K3R1K2K3R1K2R1K2S$$
$$0K1K2K3R1K2R1K2K3R1K2K3R1K2K3R1K2S$$
$$0K1K2R1K2K3R1K2K3R1K2K3R1K2K3R1K2S$$
$$0K1K2R1K2R1K2K3R1K2K3R1K2K3R1K2K3S$$
$$0K1K2K3R1K2R1K2R1K2K3R1K2K3R1K2K3S$$
$$0K1K2R1K2K3R1K2R1K2K3R1K2K3R1K2K3S$$

$$0K1K2K3R1K2K3R1K2R1K2K3R1K2K3R1K2S$$
$$0K1K2K3R1K2R1K2K3R1K2R1K2K3R1K2K3S$$
$$0K1K2R1K2K3R1K2K3R1K2R1K2K3R1K2K3S$$
$$0K1K2K3R1K2K3R1K2R1K2R1K2K3R1K2K3S$$
$$0K1K2K3R1K2K3R1K2K3R1K2R1K2K3R1K2S$$
$$0K1K2K3R1K2K3R1K2K3R1K2K3R1K2R1K2S$$

The maximum net income for years 1 to 16 equals ₦ 262,412,576.00

3.2.2 Interpretation of the First Age-Transition Diagram

Initiate the process with a new machine, perform machine replacements at age 3 with new machines for the next 12 years; thereafter perform machine replacements at age 2 for the last 4 years and then sell off the last two-year old machine at the end of the 16- year period.Similar interpretations hold for the remaining diagrams. The different age-transition diagrams reflect the multiple alternate optima and widedegree of flexibility for optimal managerial decisions.

The above results are consistent with the manual solutions in Ukwu et al. [8].

In what follows, the optimal policies and returns are now considered with respect to nonzero starting ages.

According to [7] the optimal replacement policy prescriptions and returns corresponding to the set of starting ages {1, 2,…, 10} are determined in one fell-swoop from stage 10 to 25 – a total of 16 stages starting from the top (stage 25). These are encapsulated in the age-transition diagrams startingfrom stage 10. In general, for an *n*-stage process with extended horizon length n_2, the optimal returns and age-transition diagrams are secured from stage $1+n_2-n$ to n_2.

Figure 3: Template Outputs for the Optimal Strategies and Rewards for Actual Stages 16 to 9 Using the Set of Starting Ages {1,2,3,…,10}.

Equipment Replacement Problem Solution Template					***n* Extended**	**Batch Index**	***n***		**c(0)**	**r(0)**	**s(1)**	**Factor**					
Replacement Age =			16	**years**	25	1	16		1500000	21668400	17000000	0.000001					
	Given Data (Millions)			**Stage**	25			Rates	1.2	0.9	0.98						
	***I* =**	25	*V(0) = r(0)-c(0) - I =*		-4.8316												
Age *t* (yrs.)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Revenue: *r(t)*	21.67	19.50156	17.5514	15.7963	14.216637	12.7949735	11.5155	10.364	9.327536	8.3947821	7.555304	6.79977	6.1198	5.5078	4.957	4.46133	4.01519827
Operating cost, *c(t)*	1.5	1.8	2.16	2.592	3.1104	3.73248	4.47898	5.3748	6.449725	7.7396705	9.287605	11.1451	13.374	16.049	19.26	23.1105	27.7326388
Salvage value, *s* (t)		17	16.66	16.3268	16.000264	15.6802587	15.3667	15.059	14.75813	14.462971	14.17371	13.8902	13.612	13.34	13.07	12.8119	12.5556747
K		34.36156	31.7182	29.2045	26.786496	24.4291471	22.0958	19.747	17.34078	14.828824	12.15794	9.26708	6.0858	2.5322	-1.49	-6.0935	Must Replace
R		29.1684	28.8284	28.4952	28.168664	27.8486587	27.5351	27.228	26.92653	26.631371	26.34211	26.0586	25.781	25.509	25.24	24.9803	24.7240747
Opt. value: *f*(*t*)		34.36156	31.7182	29.2045	28.168664	27.8486587	27.5351	27.228	26.92653	26.631371	26.34211	26.0586	25.781	25.509	25.24	24.9803	24.7240747
Opt. Decision		*K*	*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	24												
K		49.419764	44.595932	41.37293	38.95489596	36.59754706	34.26422	31.9157	29.509182	26.9972236	24.326337	21.43548	18.2542	14.7006	10.679	6.074874	Must Replace
R		46.52996	46.18996	45.85676	45.530224	45.21021872	44.89661	44.5893	44.288094	43.9929314	43.703672	43.4202	43.1424	42.8701	42.603	42.34187	42.08563474
Opt. value: *f*(*t*)		49.419764	46.18996	45.85676	45.530224	45.21021872	44.89661	44.5893	44.288094	43.9929314	43.703672	43.4202	43.1424	42.8701	42.603	42.34187	42.08563474
Opt. Decision		*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	23												
K		63.89152	61.248164	58.73449	56.31645596	53.95910706	51.62578	49.2773	46.870742	44.3587836	41.687897	38.79704	35.6158	32.0622	28.04	23.43643	Must Replace
R		61.588164	61.248164	60.91496	60.588428	60.26842272	59.95482	59.6475	59.346298	59.0511354	58.761876	58.4784	58.2006	57.9283	57.662	57.40008	57.14383874
Opt. value: *f*(*t*)		63.89152	61.248164	60.91496	60.588428	60.26842272	59.95482	59.6475	59.346298	59.0511354	58.761876	58.4784	58.2006	57.9283	57.662	57.40008	57.14383874
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	22												
K		78.949724	76.306368	73.79269	71.37465996	69.01731106	66.68398	64.3355	61.928946	59.4169876	56.746101	53.85524	50.674	47.1204	43.098	38.49464	Must Replace
R		76.05992	75.71992	75.38672	75.060184	74.74017872	74.42657	74.1192	73.818054	73.5228914	73.233632	72.95016	72.6724	72.4001	72.133	71.87183	71.61559474
Opt. value: *f*(*t*)		78.949724	76.306368	75.38672	75.060184	74.74017872	74.42657	74.1192	73.818054	73.5228914	73.233632	72.95016	72.6724	72.4001	72.133	71.87183	71.61559474
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	21												
K		94.007928	90.778124	88.26445	85.84641596	83.48906706	81.15574	78.8072	76.400702	73.8887436	71.217857	68.327	65.1457	61.5921	57.57	52.96639	Must Replace
R		91.118124	90.778124	90.44492	90.118388	89.79838272	89.48478	89.1774	88.876258	88.5810954	88.291836	88.00836	87.7306	87.4583	87.192	86.93004	86.67379874
Opt. value: *f*(*t*)		94.007928	90.778124	90.44492	90.118388	89.79838272	89.48478	89.1774	88.876258	88.5810954	88.291836	88.00836	87.7306	87.4583	87.192	86.93004	86.67379874
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	20												
K		108.479684	105.83633	103.3227	100.90462	98.54727106	96.21394	93.8654	91.458906	88.9469476	86.276061	83.3852	80.204	76.6503	72.628	68.0246	Must Replace
R		106.176328	105.83633	105.5031	105.176592	104.8565867	104.543	104.236	103.93446	103.639299	103.35004	103.0666	102.789	102.517	102.25	101.9882	101.7320027
Opt. value: *f*(*t*)		108.479684	105.83633	105.5031	105.176592	104.8565867	104.543	104.236	103.93446	103.639299	103.35004	103.0666	102.789	102.517	102.25	101.9882	101.7320027
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	19												
K		123.537888	120.89453	118.3809	115.962824	113.6054751	111.2721	108.924	106.51711	104.005152	101.33426	98.44341	95.2622	91.7085	87.686	83.0828	Must Replace
R		120.648084	120.30808	119.9749	119.648348	119.3283427	119.0147	118.707	118.40622	118.111055	117.8218	117.5383	117.261	116.988	116.72	116.46	116.2037587
Opt. value: *f*(*t*)		123.537888	120.89453	119.9749	119.648348	119.3283427	119.0147	118.707	118.40622	118.111055	117.8218	117.5383	117.261	116.988	116.72	116.46	116.2037587
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
Applicable State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	18												
K		138.596092	135.36629	132.8526	130.43458	128.0772311	125.7439	123.395	120.98887	118.476908	115.80602	112.9152	109.734	106.18	102.16	97.55456	Must Replace
R		135.706288	135.36629	135.0331	134.706552	134.3865467	134.0729	133.766	133.46442	133.169259	132.88	132.5965	132.319	132.046	131.78	131.5182	131.2619627
Opt. value: *f*(*t*)		138.596092	135.36629	135.0331	134.706552	134.3865467	134.0729	133.766	133.46442	133.169259	132.88	132.5965	132.319	132.046	131.78	131.5182	131.2619627
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Figure 4: Template Outputs for the Optimal Strategies and Rewards for Actual Stages 8 to 1 Using the Set of Starting Ages {1, 2, 3,... 10}

				Stage	17												
K		153.067848	150.42449	147.9108	145.492784	143.1354351	140.8021	138.454	136.04707	133.535112	130.86422	127.9734	124.792	121.239	117.22	112.6128	Must Replace
R		150.764492	150.42449	150.0913	149.764756	149.4447507	149.1311	148.824	148.52263	148.227463	147.9382	147.6547	147.377	147.105	146.84	146.5764	146.3201667
Opt. value: $f(t)$		153.067848	150.42449	150.0913	149.764756	149.4447507	149.1311	148.824	148.52263	148.227463	147.9382	147.6547	147.377	147.105	146.84	146.5764	146.3201667
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	16												
K		168.126052	165.4827	162.969	160.550988	158.1936391	155.8603	153.512	151.10527	148.593316	145.92243	143.0316	139.85	136.297	132.27	127.671	Must Replace
R		165.236248	164.89625	164.563	164.236512	163.9165067	163.6029	163.296	162.99438	162.699219	162.40996	162.1265	161.849	161.576	161.31	161.0482	160.7919227
Opt. value: $f(t)$		168.126052	165.4827	164.563	164.236512	163.9165067	163.6029	163.296	162.99438	162.699219	162.40996	162.1265	161.849	161.576	161.31	161.0482	160.7919227
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
				Stage	15												
K		183.184256	179.95445	177.4408	175.022744	172.6653951	170.3321	167.984	165.57703	163.065072	160.39418	157.5033	154.322	150.768	146.75	142.1427	
R		180.294452	179.95445	179.6213	179.294716	178.9747107	178.6611	178.354	178.05259	177.757423	177.46816	177.1847	176.907	176.635	176.37	176.1064	
Opt. value: $f(t)$		183.184256	179.95445	179.6213	179.294716	178.9747107	178.6611	178.354	178.05259	177.757423	177.46816	177.1847	176.907	176.635	176.37	176.1064	
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
				Stage	14												
K		197.656012	195.01266	192.499	190.080948	187.7235991	185.3903	183.042	180.63523	178.123276	175.45239	172.5615	169.38	165.827	161.8		
R		195.352656	195.01266	194.6795	194.35292	194.0329147	193.7193	193.412	193.11079	192.815627	192.52637	192.2429	191.965	191.693	191.43		
Opt. value: $f(t)$		197.656012	195.01266	194.6795	194.35292	194.0329147	193.7193	193.412	193.11079	192.815627	192.52637	192.2429	191.965	191.693	191.43		
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*		
State		1	2	3	4	5	6	7	8	9	10	11	12	13	14		
				Stage	13												
K		212.714216	210.07086	207.5572	205.139152	202.7818031	200.4485	198.1	195.69344	193.18148	190.51059	187.6197	184.438	180.885			
R		209.824412	209.48441	209.1512	208.824676	208.5046707	208.1911	207.884	207.58255	207.287383	206.99812	206.7146	206.437	206.165			
Opt. value: $f(t)$		212.714216	210.07086	209.1512	208.824676	208.5046707	208.1911	207.884	207.58255	207.287383	206.99812	206.7146	206.437	206.165			
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*			
State		1	2	3	4	5	6	7	8	9	10	11	12	13			
				Stage	12												
K		227.77242	224.54262	222.0289	219.610908	217.2535591	214.9202	212.572	210.16519	207.653236	204.98235	202.0915	198.91				
R		224.882616	224.54262	224.2094	223.88288	223.5628747	223.2493	222.942	222.64075	222.345587	222.05633	221.7729	221.495				
Opt. value: $f(t)$		227.77242	224.54262	224.2094	223.88288	223.5628747	223.2493	222.942	222.64075	222.345587	222.05633	221.7729	221.495				
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*				
State		1	2	3	4	5	6	7	8	9	10	11	12				
				Stage	11												
K		242.244176	239.60082	237.0871	234.669112	232.3117631	229.9784	227.63	225.2234	222.71144	220.04055	217.1497					
R		239.94082	239.60082	239.2676	238.941084	238.6210787	238.3075	238	237.69895	237.403791	237.11453	236.8311					
Opt. value: $f(t)$		242.244176	239.60082	239.2676	238.941084	238.6210787	238.3075	238	237.69895	237.403791	237.11453	236.8311					
Opt. Decision		*K*	*K/R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*					
State		1	2	3	4	5	6	7	8	9	10	11					
				Stage	10												
K		257.30238	254.65902	252.1453	249.727316	247.3699671	245.0366	242.688	240.2816	237.769644	235.09876						
R		254.412576	254.07258	253.7394	253.41284	253.0928347	252.7792	252.472	252.17071	251.875547	251.58629						
Opt. value: $f(t)$		257.30238	254.65902	253.7394	253.41284	253.0928347	252.7792	252.472	252.17071	251.875547	251.58629						
Opt. Decision		*K*	*K*	*R*	*R*	*R*	*R*	*R*	*R*	*R*	*R*						
State		1	2	3	4	5	6	7	8	9	10						

3.3 Optimal Replacement Policies

The age transition diagrams show that in any decision year, any equipment of age 3 years or above must be replaced; therefore the starting ages in the set {3, 4, . . .,16} should not be considered.

3.3.1 Age Transition Diagrams for Starting Age 1:

$$1K2K3R1K2K3R1K2K3R1K2K3R1K2R1K2K3S$$
$$1K2R1K2K3R1K2K3R1K2K3R1K2K3R1K2K3S$$
$$1K2K3R1K2R1K2K3R1K2K3R1K2K3R1K2K3S$$
$$1K2K3R1K2K3R1K2R1K2K3R1K2K3R1K2K3S$$
$$1K2K3R1K2K3R1K2K3R1K2R1K2K3R1K2K3S$$
$$1K2K3R1K2K3R1K2K3R1K2K3R1K2K3R1K2S$$

A few others are omitted. The policies call for mandatory replacement at age 3 and only once at age 2. The maximum net income for years 1 to 16 equals ₦257, 30,238.00

Note that in general, there are $2(n+1)$ concatenated age and decision symbols corresponding to the horizon length n. The above diagrams are consistent with this requirement, with $n=16$.

3.3.2 Interpretation of the Age-Transition Diagrams

The first policy starts with a year-old machine, with mandatory replacement at age 3 for the next 11 years, for two years until the beginning of year 14 when it is replaced. Finally, the last replacement machine is deployed for the last three years of the planning horizon and the sold off at the price of ₦16,326,800.00. The other diagrams can be analogously interpreted.

Starting Age 2: Unique Optimal Strategy

$$2K3R1K2K3R1K2K3R1K2K3R1K2K3R1K2K3S$$

3.3.3 Optimal Replacement Strategy

Starting with a two-year old machine, perform machine replacements only at age 3; then sell of the last machine at the end of the 16-year period for ₦16,326,800.00.

3.4 Comments on the Results of the ERO Problem

The optimal net profit is a decreasing function of the machine age.

4. SUMMARY AND CONCLUSION

The optimal policy of "Keeping or Replacing" a machine in this mining system within a sixteen-year period is to keep or replace the machine based on the age transition diagrams emanating from the solution template outputs. The equipment replacement optimization (ERO) solution templates can be adopted as the most suitable mechanism for determining the optimal replacement strategies and the corresponding net optimal profits in the replacement of electromagnetic separator machines for any horizon length between 1 and 16 years. The results can be extended to any horizon length greater than 16 provided the pertinent data are provided. The results are applicable to other mining systems.

This research work deployed computational formulas in [4] for the states corresponding to each decision year in a certain class of equipment replacement problems, thereby eliminating the drudgery and errors associated with network diagrams for such determination. The study also exploited the prototypical solution templates in [5],[6] and [7] for the determination of the optimal replacement strategies and returns to obtain the optimal policy prescriptions and the corresponding optimal net profits with respect to the case study. These policy prescriptions and the corresponding returns are encapsulated in the age transition diagrams with the corresponding returns. In general, the template could be deployed to solve each equipment replacement problem in less than 0.5 percent of the time required for the manual generation of the alternate optima.

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