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SENSITIVITY ANALYSES AND ELECTRONIC IMPLEMENTATIONS OF OPTIMAL INVESTMENT STRATEGIES AND REWARDS FOR A CERTAIN DYNAMIC CLASS OF PROBABILISTIC INVESTMENT PROBLEMS

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ABSTRACT

This article designed and implemented Excel solution templates for optimal investment strategies and the corresponding optimal rewards, for the largest class of certain probabilistic dynamic investment problems for practical and realistic consideration, using backward recursive dynamic programming. It went further to optimally deploy the templates for sensitivity analysis of the problem, in a matter of minutes. These activities could hardly be contemplated in manual computations. The templates reflected and demonstrated consistency with the base results.

KEYWORDS: Dynamic Class, Dynamic Programming, Electronic Implementation, Market Conditions, Optimal Investment Strategy, Policy Prescriptions, Probabilistic Investment Problems, Recursions, Sensitivity Analysis, Solution Templates.

1. Introduction

Probabilistic dynamic programming is characterized by the uncertainty of states and returns at each stage. It arises for the most part in the treatment of stochastic inventory models and in Markovian decision processes. However, this investigation will focus on the following class of stationary investment problems, as formulated but not proved by Taha [1]. Review of literature reveals the nonexistence of any formal proof of the prescribed investment strategy in [1]; needless to say that no extension of the formulation has been attempted by any author, until

Ukwu [2], which filled these yawning gaps. [2] proved the results in [1] and obtained nontrivial extensions of those results to the dynamic class of investment problems, thus adding to the existing body of knowledge. However, the computing complexity imposed by dynamic programming recursions for large problem instances makes electronic implementation of optimal policy prescriptions imperative, subject to its feasibility. Unfortunately there is currently no software for solving this class of dynamic programming problems, and indeed dynamic programming problems in general. Worse still, implementation issues are hardly addressed in most operations research texts and articles. Ukwu [3, 4] provided the much desired implementation paradigm shift by designing and implementing Excel solution templates for equipment replacement problems based on decision years and corresponding sets of feasible ages. Sequel to these, Ukwu [5] obtained novel formulations and prototypical solution template for time-based recursions of equipment replacement problems with stationary pertinent data; Ukwu [6] performed sensitivity analysis of time horizon for a class equipment replacement problems with stationary pertinent data.

This article takes up yet the challenge of designing and digitally implementing the optimal results in [2]. The class of problems solved in [2] is restated below as follows.

2. Materials and Methods

2.1 Dynamic Investment Problem with Uncertainty

An individual wishes to invest up to C dollars in the stock market over the next n (years or periods). The investment plan calls for buying the stock at the start of the year (period) and selling it off at the end of the same year (period). Accumulated money may then be reinvested (in whole or part) at the start of the following year (period). The degree of risk in the investment is represented by expressing the return probabilistically. A study of the market shows that the return on investment is affected by m_i (favourable or unfavourable) market conditions and that condition i yields a return r_{k_i} with probability $p_{k_i}, i \in \{1,2,\ldots,n\}, k_i \in \{1,2,\cdots,m_i\}$.

How should the amount C be invested to realize the highest capital accumulation at the end of n time periods? Prove that the prescribed investment strategy is optimal.

2.2 Definition of investment capacities and decision variables

$x_i =$ Amount of funds available for investment at the start of period i. Note that $x_i = C$.
$y_i =$ Amount actually invested at the start of period i. Clearly, $y_i \leq x_i$.

2.3 Elements of the Dynamic Programming Model

1. Stage i is represented by period i.
2. The alternatives at stage i are given by y_i.
3. The state at stage i is given by x_i.

2.4 Definition of the Backward Dynamic Programming Recursions

Let $f_i(x_i) =$ maximum expected funds for periods (years) $i, i+1, \ldots,$ and n, given x_i at the start of period i.

For market condition k we have the following relationship between stages i and $i+1$

a) $$x_{i+1} = \underbrace{x_i - y_i}_{\substack{\text{unutilized funds in period } i \\ \text{(carried over to period } i+1)}} + \underbrace{(1 + r_{k_i}) y_i}_{\text{return on the 1-period investment}}$$
$$= x_i + r_{k_i} y_i$$

Given that market condition k_i occurs with probability $P_{k_i}; k_i \in \{1, 2, \ldots, m_i\}$ (m_i market contigencies), let $f_i(x_i)$ be the maximal expected funds, for periods $i, i+1, \ldots, n$, given that x_i monetary amount is available for investment at the start of period i.

3. Results and Discussion

3.1 Theorem 2: The Optimal Policy Prescription

For the general investment problem with an arbitrary number of different market conditions for each period and corresponding returns, define the following:

$m_i =$ Number of market conditions in year i

$n =$ Horizon length

$r_{k_i} =$ Market return for market condition k_i in period i (stage i)
$p_{k_i} =$ Probability of market condition k_i in period i

$$\bar{r}_i = \sum_{k_i=1}^{m_i} p_{k_i} r_{k_i}, i \in \{1,\cdots,n\}; \bar{R}_i = \{\bar{r}_i, \bar{r}_{i+1},\cdots,\bar{r}_n\}, i \in \{1,2,\cdots,n-1\}; \bar{R}_n = \bar{r}_n;$$

$$\bar{R}_i^+ = \{\bar{r} \in \bar{R}_i : \bar{r} > 0\}; \bar{R}_i^- = \{\bar{r} \in \bar{R}_i : \bar{r} \le 0\}. \ \left(\text{Clearly } \bar{R}_i = \bar{R}_i^- \cup \bar{R}^+ \text{ and } \bar{R}_i^- \cap \bar{R}^+ = \varnothing\right)$$

Then subject to the standing hypotheses, the optimal investment strategy and the corresponding optimal return are prescribed as follows:

(a) $y_i^* = x_i \operatorname{sgn}\left(\max\{\bar{r}_i, 0\}\right), i \in \{1,2,\cdots,n\}$

(b) $f_i(x_i) = \prod_{\bar{r} \in \bar{R}_i^+} (1+\bar{r}) x_i$, if $\bar{R}_i^+$ is nonempty

(c) $f_i(x_i) = x_i$, if if $\bar{R}_i^+$ is a null set $\left(\text{Equivalently } \bar{R}_i = \bar{R}_i^-\right)$,

where y_i^* is the optimal investment strategy at the start of period $i; i \in \{1,2,\ldots,n\}$. $f_i(x_i)$ is the maximal expected funds for periods $i, i+1,\ldots,n$, given that the amount x_i is available for investment at the start of period i. Cf. [2], for proof.

The solution template for the optimal electronic implementation of the policy prescription is routed in the appropriate Excel code and interface design, as exposed below.

3.2 Notations and Features of the Design Template Below:

1. Identifiers are written in bold typeface while numeric values are not bolded; num_val is the numeric value assigned to **num** and stored at the cell location.Space constraints may necessitate writing just 'val' instead of 'num_val' at the cell location adjacent to and to the right of num. **Opt.** stands for **Optimal;Dec.** stands for **Decision.**
2. Formulas are preceded by ' = '
3. Following the execution of a formula by the keyboard operation **'< Enter > '**, the act of clicking back on a specified cell, positioning the cursor at the right edge of the cell until a crosshair appears and dragging the cross-hair horizontally or vertically to some cell will be referred to as clerical routine/duty.
4. = (h) or $(.) \rightarrow$ represents cross-hair horizontal dragging to the right, with the typed formula indicated by =(.); (v) or $=(.)\downarrow$ represents cross-hair vertical dragging downwards.
5. Numbering of formulas indicates order of execution and numbering appended with "h" or "v" indicates cross-hair horizontal or verticaldragging routine. For example, = (1v)

indicates that the first unspecified formula should be typed at that cell location and cross-hair downward vertical dragging routine implemented.

6. The coefficient of x_i in the optimal decision y_i^* is either 0 or 1. Also $f_i(x_i)$ is a linear function of x_i. Therefore for each $i \in \{n, n-1, \cdots, 1\}$, $f_i(x_i)$ and y_i^* are completely determined from the listing of their respective x_i coefficients. Let these be denoted by a_i and b_i respectively.

   Then, in particular,

$$b_i = \begin{cases} 1, \text{ if } y_i^* = x_i \\ 0, \text{ if } y_i^* = 0 \end{cases}$$

7. The optimal funds accumulation at the end of the planning horizon is simply $\$f_1(x_1)$.

For template clarity, the following notations are equivalent:

$$r_{k_i}, k_i \in \{1, 2, \cdots, m_i\} \Leftrightarrow r_{ik}, k \in \{1, 2, \cdots, m_i\}; p_{k_i}, k_i \in \{1, 2, \cdots, m_i\} \Leftrightarrow p_{ik}, k \in \{1, 2, \cdots, m_i\}$$

Table 1: Prototypical Excel Solution Template Design andSensitivity Analysis for the Dynamic Class of Probabilistic Investment Problems

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1		***N***	16														
2		Market Conditions						Market Conditions									
3	**k**	1	2	3	4	5	6	1	2	3	4	5	6			**Opt.Dec**	**Opt. Obj**
4						***C***	val									**Coeff.**	**Coeff**
5	**Year i**	**Market Returns r_{ik}**						**Return Probabilities p_{ik}**						$\bar{r}_i$	$1+\bar{r}_i$	b_i	a_i
6															1		
7	=c1	0	1	1.5				.35	.2	.2	.2	.02	.03	=(1v)	= (2v)	= (3v)	=(4v)
8	15	-1	.5	.4				.3	.2	.1	.1	.15	.15				
9	14	1.5	1	.6				.3	.1	.2	.2	.12	.08				
10	13	.9	.7	.2	-1	1		.15	.2	.2	.3	.1	.05				
11	12	3	-1					.05	.2	.2	.2	.1	.25				
12	11	.7	.5	1	-1			.1	.2	.2	.2	.15	.15				
13	10	.8	.4	.2	.1	0		.3	.2	.2	.2	.04	.06				
14	9	4	-1	-1				.02	.14	.24	.3	.12	.18				
15	8	1	0	-1	.4			.3	.2	.1	.15	.1	.15				
16	7	2	1	.5	-1			.1	.3	.2	.25	.07	.08				
17	6	1.2	-1	-1	1	.8		.2	.1	.1	.1	.2	.3				
18	5	1	0	1	.5	.6		.1	.3	.2	.25	.1	.05				
19	4	1	1.2	0	1.4	1	1	.1	.1	.2	.1	.2	.3				
20	3	1	.5	-.5	-.4	0		.05	.2	.2	.2	.1	.25				
21	2	.6	.5	2	-1	0	.2	.35	.2	.2	.2	.02	.03				
22	1	.4	.3	1.5	-.6	.8	1	.15	.22	.12	.33	.1	.08				

3.3 An Exposition on the Solution Template

Step 1

Rearrange given pertinent data to start from the terminal stage (terminal year/period *n*) and end at stage 1

Use Excel references A1:Q5, for documentation, and identifiers. Save the problem data in the indicated cells: A1:M20.

Store the horizon length, $n,$ in cell reference C1

Store the value of the identifier $\boldsymbol{C}$ in cell $G4$ and note that

$x_1 =$ (the value of $C,$ stored in G4).

Step2

Type the identifier $\bar{\boldsymbol{r}}_i$ for the average market return in year i in cell N5 and the yield $1+\bar{r}_i$ in cell O5

Type the identifier $\boldsymbol{b}_i$ for the coefficient of x_i, with respect to the optimal decision variable y_i^*, in stage i , in the cell P5, noting that $y_i^* = b_i x_i$.

Type the identifier $\boldsymbol{a}_i$ for the coefficient of x_i, with respect to the stage i optimal function value $f_i(x_i)$ in cell Q5, noting that $f_i = a_i x_i$.

Type 1 in the cell reference Q6.

Step 3

Implement the stage numbering by first executing typing '= C1', in A7, followed by **<Enter>;** then

type ' = $A7 – 1' in cell A8, followed by the implementation of the cross-hair downward vertical dragging routine, terminating at '1'..

Step 4

Type the following code segments without the quotes and commas, in O7, P7, and Q7 respectively

'= Sumproduct($C7:$H7, $I7:$N7)',' = 1 + $O7', ' = If ($O7 > 0, 1, 0)', followed by the implementation of the cross-hair downward vertical dragging routine, to secure the values of

$\overline{r}_i$, $1+\overline{r}_i$, and b_i, for years n to 1.

Alternatively, after typing the code segments in the indicated three cells, select all three cells and implement the cross-hair downward vertical dragging routine, to secure the values of $\overline{r}_i$, $1+\overline{r}_i$, and b_i,

for years n to 1, in one fell swoop.

Step 5

(i) Type ‘ =If (P7 > 1, P6*P7, P6)’, in cell R7, followed by <**Enter,** to secure a_n**.** STOP**!**

(ii) Type ‘ = If($P8 > 1, $R7*$P8, $R7)’, in cell R8, followed by the implementation of the cross-hair downward vertical dragging routine, to secure the values

$a_{n-1},\ a_{n-2},\cdots,a_1.$

(iii) Type the identifier ‘**Optimal objective value =** ‘in the merged cells B24:D24.

(iv) Generate the optimal objective value automatically, as in cell reference E24, by typing

‘ = G4*$R22’, followed by the execution of the <**Enter**> operation.

The following table provides the pertinent data and the corresponding optimal solution and rewards for a problem instance with $n=16;\min\left\{m_1,m_2,\cdots,m_{16}\right\}=2,\ \max\left\{m_1,m_2,\cdots,m_{16}\right\}=6;$ $C=10,000;\ r_{ik},k\in\left\{1,\cdots,m_i\right\};\ p_{ik},k\in\left\{1,\cdots,m_i\right\}.$

A B C D E F G H I J K L M N O P Q R

1	n =	16															
2		**Market Conditions**						**Market Conditions**									
3	***k***	1	2	3	4	5	6	1	2	3	4	5	6			**Opt. Dec. coeff.**	**Opt. obj. coeff**
4					***C*** =	10000											
5	**Year i**	**Market Returns** r_{ik}						**Market Return Probabilities** p_{ik}						$\bar{r}_i$	$1+\bar{r}_i$	b_i	a_i
6															1		
7	16	0	1	1.5				0.25	0.3	0.2	0.2	0.02	0.03	0.6	1.6	**1**	**1.6**
8	15	-1	0.5	0.4				0.3	0.2	0.1	0.1	0.15	0.15	-0.16	0.84	**0**	**1.600000**
9	14	1.5	1	0.6				0.3	0.1	0.2	0.2	0.12	0.08	0.67	1.67	**1**	**2.672000**
10	13	0.9	0.7	0.2	-1	1		0.15	0.2	0.2	0.3	0.1	0.05	0.115	1.115	**1**	**2.979280**
11	12	3	-1					0.05	0.2	0.2	0.2	0.1	0.25	-0.05	0.95	**0**	**2.979280**
12	11	0.7	0.5	1	-1			0.1	0.2	0.2	0.2	0.15	0.15	0.17	1.17	**1**	**3.485758**
13	10	0.8	0.4	0.2	0.1	0		0.3	0.2	0.2	0.2	0.04	0.06	0.38	1.38	**1**	**4.810345**
14	9	4	-1	-1				0.02	0.14	0.24	0.3	0.12	0.18	-0.3	0.7	**0**	**4.810345**
15	8	1	0	-1	0.4			0.3	0.2	0.1	0.15	0.1	0.15	0.26	1.26	**1**	**6.061035**
16	7	2	1	0.5	-1			0.1	0.1	0.2	0.45	0.08	0.07	-0.05	0.95	**0**	**6.061035**
17	6	1.2	-1	-1	1	0.9		0.1	0.3	0.3	0.15	0.05	0.05	-0.285	0.715	**0**	**6.061035**
18	5	1	0	1	0.5	0.6		0.1	0.3	0.2	0.25	0.1	0.05	0.485	1.485	**1**	**9.000637**
19	4	1	1.2	0	1.4	1	1	0.1	0.1	0.2	0.1	0.2	0.3	0.86	1.86	**1**	**16.741186**
20	3	1	0.5	-0.5	-0.4	0		0.05	0.2	0.2	0.2	0.1	0.25	-0.03	0.97	**0**	**16.741186**
21	2	0.6	0.5	1	-1	0	0.2	0.1	0.2	0.2	0.45	0.02	0.03	-0.084	0.916	**0**	**16.741186**
22	1	0.4	0.3	1.5	-0.6	0.8	1	0.15	0.22	0.12	0.33	0.1	0.08	0.268	1.268	**1**	**21.227823**
	Optimal objective value =			**$212,278.23**													

Figure 1: A Snapshot of Excel Outputs of the Optimal Investment Strategy and Rewards

Therefore, the optimal objective value is $212, 278.23

3.4 Optimal Investment Policy Prescription

Invest all available funds at the beginning of years 1, 4, 5, 8, 10, 11, 13, 14, and 16 and none at all, at the beginning of the other years. The expected accumulated funds at the end of the 16 years $= f_1(x_1) = \$212,278.23$.

The next and final table provides the pertinent data and the corresponding optimal solution and rewards for a problem instance with $n = 50; \min\{m_1, m_2, \cdots, m_{50}\} = 2, \max\{m_1, m_2, \cdots, m_{50}\} = 6;$ $C = 10,000; r_{ik}, k \in \{1, \cdots, m_i\}; p_{ik}, k \in \{1, \cdots, m_i\}.$

n =	50														Opt. Dec.	Opt. obj.
				C =	10000										coeff.	coeff
Year i	Market Returns r_{ik}						Market Return Probabilities p_{ik}						$\bar{r}_i$	$1+\bar{r}_i$	b_i	a_i
														1		
50	0	1	1.5				0.35	0.2	0.2	0.2	0.02	0.03	0.5	1.5	**1**	**1.5**
49	-1	0.5	0.4				0.3	0.2	0.1	0.1	0.15	0.15	-0.16	0.84	**0**	**1.500000**
48	1.5	1	0.6				0.3	0.1	0.2	0.2	0.12	0.08	0.67	1.67	**1**	**2.505000**
47	0.9	0.7	0.2	-1	1		0.15	0.2	0.2	0.3	0.1	0.05	0.115	1.115	**1**	**2.793075**
46	3	-1					0.05	0.2	0.2	0.2	0.1	0.25	-0.05	0.95	**0**	**2.793075**
45	0.7	0.5	1	-1			0.1	0.2	0.2	0.2	0.15	0.15	0.17	1.17	**1**	**3.267898**
44	0.8	0.4	0.2	0.1	0		0.3	0.2	0.2	0.2	0.04	0.06	0.38	1.38	**1**	**4.509699**
43	4	-1	-1				0.02	0.14	0.24	0.3	0.12	0.18	-0.3	0.7	**0**	**4.509699**
42	1	0	-1	0.4			0.3	0.2	0.1	0.15	0.1	0.15	0.26	1.26	**1**	**5.682221**
41	2	1	0.5	-1			0.1	0.1	0.2	0.45	0.08	0.07	-0.05	0.95	**0**	**5.682221**
40	1.2	-1	-1	1	0.8		0.1	0.3	0.3	0.15	0.05	0.05	-0.29	0.71	**0**	**5.682221**
39	1	0	1	0.5	0.6		0.1	0.3	0.2	0.25	0.1	0.05	0.485	1.485	**1**	**8.438098**
38	1	1.2	0	1.4	1	1	0.1	0.1	0.2	0.1	0.2	0.3	0.86	1.86	**1**	**15.694862**
37	1	0.5	-0.5	-0.4	0		0.05	0.2	0.2	0.2	0.1	0.25	-0.03	0.97	**0**	**15.694862**
36	0.6	0.5	1	-1	0	0.2	0.1	0.2	0.2	0.45	0.02	0.03	-0.084	0.916	**0**	**15.694862**
35	0.4	0.3	1.5	-0.6	0.5	1	0.15	0.22	0.12	0.33	0.1	0.08	0.238	1.238	**1**	**19.430239**
34	0	1	1.5				0.35	0.2	0.2	0.2	0.02	0.03	0.5	1.5	**1**	**29.145358**
33	-1	0.5	0.4				0.3	0.2	0.1	0.1	0.15	0.15	-0.16	0.84	**0**	**29.145358**
32	1.5	1	0.6				0.3	0.1	0.2	0.2	0.12	0.08	0.67	1.67	**1**	**48.672748**
31	0.9	0.7	0.2	-1	1		0.15	0.2	0.2	0.3	0.1	0.05	0.115	1.115	**1**	**54.270114**
30	3	-1					0.05	0.2	0.2	0.2	0.1	0.25	-0.05	0.95	**0**	**54.270114**
29	0.7	0.5	1	-1			0.1	0.2	0.2	0.2	0.15	0.15	0.17	1.17	**1**	**63.496033**
28	0.8	0.4	0.2	0.1	0		0.3	0.2	0.2	0.2	0.04	0.06	0.38	1.38	**1**	**87.624525**
27	4	-1	-1				0.02	0.14	0.24	0.3	0.12	0.18	-0.3	0.7	**0**	**87.624525**
26	1	0	-1	0.4			0.3	0.2	0.1	0.15	0.1	0.15	0.26	1.26	**1**	**110.406902**
25	2	1	0.5	-1			0.1	0.1	0.2	0.45	0.08	0.07	-0.05	0.95	**0**	**110.406902**
24	1.2	-1	-1	1	0.8		0.1	0.3	0.3	0.15	0.05	0.05	-0.29	0.71	**0**	**110.406902**
23	1	0	1	0.5	0.6		0.1	0.3	0.2	0.25	0.1	0.05	0.485	1.485	**1**	**163.954250**
22	1	1.2	0	1.4	1	1	0.1	0.1	0.2	0.1	0.2	0.3	0.86	1.86	**1**	**304.954904**
21	1	0.5	-0.5	-0.4	0		0.05	0.2	0.2	0.2	0.1	0.25	-0.03	0.97	**0**	**304.954904**
20	0.6	0.5	1	-1	0	0.2	0.1	0.2	0.2	0.45	0.02	0.03	-0.084	0.916	**0**	**304.954904**
19	0.4	0.3	1.5	-0.6	0.5	1	0.15	0.22	0.12	0.33	0.1	0.08	0.238	1.238	**1**	**377.534172**
18	1	0.5	-0.5	-0.4	0		0.05	0.2	0.2	0.2	0.1	0.25	-0.03	0.97	**0**	**377.534172**
17	0.6	0.5	1	-1	0	0.2	0.1	0.2	0.2	0.45	0.02	0.03	-0.084	0.916	**0**	**377.534172**
16	0.4	0.3	1.5	-0.6	0.5	1	0.15	0.22	0.12	0.33	0.1	0.08	0.238	1.238	**1**	**467.387304**
15	4	-1	-1				0.02	0.14	0.24	0.3	0.12	0.18	-0.3	0.7	**0**	**467.387304**
14	0.8	0.4	0.2	0.1	0		0.3	0.2	0.2	0.2	0.04	0.06	0.38	1.38	**1**	**644.994480**
13	4	-1	-1				0.02	0.14	0.24	0.3	0.12	0.18	-0.3	0.7	**0**	**644.994480**
12	1	0	-1	0.4			0.3	0.2	0.1	0.15	0.1	0.15	0.26	1.26	**1**	**812.693045**
11	2	1	0.5	-1			0.1	0.1	0.2	0.45	0.08	0.07	-0.05	0.95	**0**	**812.693045**
10	1.2	-1	-1	1	0.8		0.1	0.3	0.3	0.15	0.05	0.05	-0.29	0.71	**0**	**812.693045**
9	1	0	1	0.5	0.6		0.1	0.3	0.2	0.25	0.1	0.05	0.485	1.485	**1**	**1206.849172**
8	1	1.2	0	1.4	1	1	0.1	0.1	0.2	0.1	0.2	0.3	0.86	1.86	**1**	**2244.739459**
7	1	0.5	-0.5	-0.4	0		0.05	0.2	0.2	0.2	0.1	0.25	-0.03	0.97	**0**	**2244.739459**
6	0.6	0.5	1	-1	0	0.2	0.1	0.2	0.2	0.45	0.02	0.03	-0.084	0.916	**0**	**2244.739459**
5	0.4	0.3	1.5	-0.6	0.5	1	0.15	0.22	0.12	0.33	0.1	0.08	0.238	1.238	**1**	**2778.987451**
4	0	1	1.5				0.35	0.2	0.2	0.2	0.02	0.03	0.5	1.5	**1**	**4168.481176**
3	-1	0.5	0.4				0.3	0.2	0.1	0.1	0.15	0.15	-0.16	0.84	**0**	**4168.481176**
2	1.5	1	0.6				0.3	0.1	0.2	0.2	0.12	0.08	0.67	1.67	**1**	**6961.363564**
1	0.9	0.7	0.2	-1	1		0.15	0.2	0.2	0.3	0.1	0.05	0.115	1.115	**1**	**7761.920373**

Figure 2: A Snapshot of Excel Outputs of the Optimal Investment Strategy and Rewards for large horizon Lengths and for problem sensitivity Analyses

Therefore, the optimal objective value is $77, 619,203.73

3.5 Optimal Investment Policy Prescription

Invest all available funds at the beginning of years 1, 2, 4, 5, 8,9, 12, 14, 16 , 19, 22, 23, 26, 28, 29, 31, 32, 34, 35, 38, 39, 42, 44, 45, 47, 48 , 50 and none at all, at the beginning of the other years. The expected accumulated funds at the end of the 50 years $= f_1(x_1) = \$77,619,203.73$

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3.6 On problem Sensitivity Analyses

The horizon length of 50 is substantial enough to encourage sensitivity analyses, based on $2 \le n \le 50$, for fixed m_i and varying C, r_{ik} and p_{ik}. In particular, for the case $n = 50$, the generation of the generation of the optimal solutions is immediate, without adjustments in the rows or columns.

For $\max\{m_1, m_2, \cdots, m_n\} \ge 7$, one need only insert the appropriate number of columns to the right of the last columns for r_{ik} and p_{ik}, supply additional data and update the code column for $\bar{r}_i, 1+\bar{r}_i$, b_i, and a_i appropriately. This exercise should consume no more than 2 minutes. If $n \ge 51$, all that is required is to increase the number of rows approximately, supply required additional data and then implement the crosshair-dragging routine from last cell values of $\bar{r}_i, 1+\bar{r}_i$, b_i, and a_i. Evidently, there is no restriction on problem size, noting that the dimensional limitations in Excel can hardly be exhausted.

4. Conclusion

This article appropriated the analytic structure of the optimal investment strategy formulated and proved in [2], to deftly develop Excel code and design corresponding Excel spreadsheet template for the electronic implementation of the optimal policy prescription for a general class of probabilistic investment problems of the dynamic variety. It proceeded to give a complete exposition on the template and provided an illustrative problem that demonstrated the functionality and tremendous power of the solution template, as well as performed sensitivity analysis on a large problem instance. The results are quite robust and unprecedentedand the capacity for their implementations subject only to Excel functionality and size limitation of 170 factorial- approximately 7.2574E+306 (in Scientific notation). This requirement can hardly be exhausted or violated in most practical problems. The solution template clearly defines much desiredrapid solution implementation paradigm shift and problem sensitivity analyses hitherto unfathomable and too computationally prohibitive to be contemplated.

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