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A New [14 8 3]-Linear Code From the Aunu Generated [7 4 2] -Linear Code and the Known [7 4 3] Hamming Code Using the (U|U+V) Construction

Ibrahim AA[1]*, Chun PB[2] and Kamoh NM[3]

[1]Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria
[2]Department of Mathematics, Plateau State University Bokkos, Jos, Nigeria
[3]Department of Mathematics, Bingham University, Karu, Nigeria

Abstract

In this communication, we enumerate the construction of a [7 4 2]- linear code which is an extended code of the [6 4 1] code and is in one-one correspondence with the known [7 4 3] - Hamming code. Our construction is due to the Carley table for n=7of the generated points of was permutations of the (132) and (123)-avoiding patterns of the non-associative AUNU schemes. Next, [7 4 2] linear code so constructed is combined with the known Hamming [7 4 3] code using the (u|u+v)-construction to obtain a new hybrid and more practical single [14 8 3] error- correcting code.

Keywords: Cayley tables; AUNU scheme; Hamming codes; Standard generator matrix; Extended code ; Reduced Row Echelon form (RREF); [n k d] linear code; (u|u+v) construction; Parity check matrix

Introduction

Historically, Claude Shannon's paper titled "A Mathematical theory of Communication" in the early 1940s signified the beginning of coding theory and the first error-correcting code to arise was the presently known Hamming [7,4,3] code, discovered by Richard Hamming in the late 1940s [1]. As it is central, the main objective in coding theory is to devise methods of encoding and decoding so as to effect the total elimination or minimization of errors that may have occurred during transmission [2] due to disturbances in the channel. The special class of the (132) and (123) avoiding Patterns of AUNU permutations has found applications in various areas of applied Mathematics [3]. The authors had reported the application of the adjacency matrix of Eulerian graphs due to the (132) - avoiding patterns of AUNU numbers in the generation and analysis of some classes of linear and cyclic codes [4,5], respectively. The authors utilized the Carley tables for n=5 [6] to derive a standard form of the generator/parity check matrix for some code. In this article [7], we enumerate the construction of a [7 4 2] – linear Code from the Carley table for n=7 of the generated points of was permutations of the (132) and (123) avoiding patterns of the non-commutative AUNU schemes [8]. The [742] linear code is then shown to be an extended [9] code of the [6 4 1] code and is in one-one correspondence with the [7 4 3]- Hamming Code. Moreover, the [7 4 2]-linear code so generated is [10,11] then combined with the known Hamming [7 4 3] code using the (u|u+v) construction method to obtain a new and more Practical single error correcting code with dimensions [12] n=14, k=8 and d=3.

Some Basic Concepts

Generator matrix

A generator matrix G for a linear code C is a kXn matrix for which the rows are a basis for C. If G is a generator matrix for C, then $C=\left\{aG \middle| a\in F^k\right\}$. G is said to be in standard form(often called the Reduce Echelon form) if $G=(I_k|X)$ where I_k is the $k\times k$ identity matrix.

The $(U|U+V)$ construction

Two codes of the same length can be combined to form a third code twice the length in a way similar to the direct sum of the codes construction. This is achieved as follows;

Let C_i be an $[n,k_i,d_i]$ code for i∈{1,2}, both over the same finite field F_q. The $(u|u+v)$ construction produces a $[2n,k_1+k_2,\min(2d_1,d_2)]$ linear code

$$C=\{(u,u+v)\,|\,u\in C_1, v\in C_2\} \tag{1}$$

Remark

If C_i is a linear code and has generator matrix G_i and parity check matrix H_i, then the new code C as defined in (1) above has generator and parity check matrices as,

$$G=\begin{bmatrix} G_1 & G_1 \\ 0 & G_2 \end{bmatrix} \text{ and } H=\begin{bmatrix} H_1 & 0 \\ -H_2 & H_2 \end{bmatrix} \text{ respectively.}$$

Since the minimum distance of the direct sum of two codes does not exceed the minimum distance of either of the codes, then it is of little use in applications and is primarily of theoretical interest. As such, we for the purpose of this research concentrate on the $(u|u+v)$ construction.

Example 1

Consider the binary [8,4,4] binary code C with generator matrix

***Corresponding author:** Ibrahim AA, Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria, Tel: +2348035929091; E-mail: chunpamson@gmail.com

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$$G=\begin{bmatrix}1&0&1&0&1&0&1&0\\0&1&0&1&0&1&0&1\\0&0&1&1&0&0&1&1\\0&0&0&0&1&1&1&1\end{bmatrix},$$

Then C can be produced by combining the [4,3,2] code C_1 and the [4,1,4] code C_2 with generator matrices $G_1=\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&0&1&1\end{bmatrix}$ and G_2[1 1 1 1] respectively, using the $(u|u+v)$ construction.

Methodology

Cayley tables

We consider the Carley table below, which is constructed using $A_n(132)$ for n=7 [2]

We now convert the entries of the Carley table above to the binary system using Modulus 2 arithmetic. Table 1, thus becomes;

The above table is the matrix G below;

$$G=\begin{bmatrix}1&1&1&1&0&0&0\\1&0&1&1&0&0&1\\1&1&0&0&1&1&0\\1&0&0&0&1&1&1\\1&1&0&1&0&1&0\end{bmatrix}$$

Clearly, all possible linear combinations of the rows of G generates a linear Code say C of length n=7 and size M=16 with the following Code words;

0 0 0 0 0 0 0	0 0 1 1 1 1 0	1 0 0 1 0 1 1	1 1 0 0 1 1 0
0 1 0 0 0 0 1	0 1 0 1 1 0 1	1 0 0 0 1 1 1	1 1 0 1 0 1 0
0 0 1 0 0 1 0	0 1 1 0 0 1 1	1 0 1 1 0 0 1	1 1 1 0 1 0 0
0 0 0 1 1 0 0	0 1 1 1 1 1 1	1 0 1 0 1 0 1	1 1 1 1 0 0 0

Now, since G has five rows and generates a code with sixteen code words, we seek a generator matrix for G. Deleting the first row and last column of G, we obtain a Matrix say $G^{II}=\begin{bmatrix}1&0&1&1&0&0\\1&1&0&0&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}$

Next, we apply the following series of row operations on G^{II}, we have;

Θ	1	2	3	4	5	6	7
1	1	3	5	7	2	4	6
2	1	4	7	3	6	2	5
3	1	5	2	6	3	7	4
4	1	6	4	2	7	5	3
5	1	7	6	5	4	3	2

Θ	1	2	3	4	5	6	7
1	1	1	1	1	0	0	0
2	1	0	1	1	0	0	1
3	1	1	0	0	1	1	0
4	1	0	0	0	1	1	1
5	1	1	0	1	0	1	0

Table 1: Carley table for n=7 showing generated points of was permutations of (132) and (123)-avoiding patterns of AUNU scheme under the action of Θ.

1. $R_2=R_1+R_2$
2. $2\text{-}R_3=R_1+R_3$
3. $R_2=R_3+R_2$
4. $R_4=R_1+R_4$
5. $R_4=R_4+R_2$
6. $R_4=R_3+R_4$
7. $R_1=R_1+R_3$
8. $R_3=R_4+R_3$

Respectively as follows

$$G^{II}=\begin{bmatrix}1&0&1&1&0&0\\1&1&0&0&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}\to\begin{bmatrix}1&0&1&1&0&0\\0&1&1&1&1&1\\1&0&0&0&1&1\\1&1&0&1&0&1\end{bmatrix}\to\begin{bmatrix}1&0&1&1&0&0\\0&1&1&1&1&1\\0&0&1&1&1&1\\1&1&0&1&0&1\end{bmatrix}$$

$$\to\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\1&1&0&1&0&1\end{bmatrix}\to\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&1&1&0&0&1\end{bmatrix}\to\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&1&0&0&1\end{bmatrix}$$

$$\to\begin{bmatrix}1&0&1&1&0&0\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&0&1&1&0\end{bmatrix}\to\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&1&1&1\\0&0&0&1&1&0\end{bmatrix}\to\begin{bmatrix}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&0&0&1\\0&0&0&1&1&0\end{bmatrix}$$

$$=\left[\begin{array}{cccc|cc}1&0&0&0&1&1\\0&1&0&0&0&0\\0&0&1&0&0&1\\0&0&0&1&1&0\end{array}\right]=G^{I}$$

Observe that G^I above is a generator matrix in standard form [1], $i.e\ G^I=(I_k|X_{k\times n-k})$, but G^I which is a generator matrix for C has words (rows) of length n=6, while C has word length n=7. Note also that all words in C has even weight, therefore extending the rows of G^I by adding one digit each, to make the number of nonzero coordinates in each row of G^I even, we obtain;

$$G^{I}=\left[\begin{array}{cccccc|c}1&0&0&0&1&1&1\\0&1&0&0&0&0&1\\0&0&1&0&0&1&0\\0&0&0&1&1&0&0\end{array}\right]$$

Which is the required generator matrix for our [7 4 2]-linear code C.

The $(U|U+V)$ construction

In what follows, we combined the [7 4 2]-linear code constructed [3] with the known Hamming [7 4 3] code to obtain a Hybrid single error correcting linear code.

Now, the [7 4 2]-linear code has the following parameters; n=7, k=4, d=2 and the known [7 4 3]-Hamming code has parameters; n=7, k=4, d=3.

On combining the two codes using the $(u|u+v)$ construction, we shall obtain a

$[2(n=7),(k_1=4+k_2=4), \min(2(d_1=2), (d_2=3)]$

$=[2(7), 4+4, \min(2(2), 3)]$

$=[14, 8, 3]$ - Linear code

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Firstly, the [7 4 2]- linear code has generator and parity check matrices G_1 and H_1 as follows;

$$G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \; and \; H_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

While for the generalize Hamming binary [7 4 3]-linear code, its generator and parity check matrices G_1 and H_2 are;

$$G_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \; and \; H_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

respectively.

Now, by the $(u|u+v)$ construction, our new code which is as a result of the combination of the above codes shall be a [14 8 3] – linear code whose generator and parity check matrices G_0 and H_0 are;

$$G_O = \begin{bmatrix} G_1 & G_1 \\ 0 & G_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

and

$$H_O = \begin{bmatrix} H_1 & 0 \\ -H_2 & H_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

As such, a single error correcting code of length n=14, dimension k=8 and Hamming distance d=3 has been constructed.

Findings

The Carley table for n=7 of the generated points of the (132) and (123)-avoiding patterns of the non-associative AUNU schemes has been used to construct a [742]-linear code C which is an extended code of the [641] code. Moreover, the [742]-linear code so generated is then utilized using the (u|u+v) construction method to obtain a new and more practical single error correcting code with dimensions n=14, k=8 and d=3.

Conclusion

This paper has further pointed the applicability of the AUNU schemes in the direction of coding theory, i.e., Codes, generation and analysis.

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