

BLACK ELECTROMAGNETIC HOLE

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Abstract - In this paper, the general mechanical equation of motion for photons moving in general gravitational field is derived and solved. The equation predicts the existence of black electromagnetic hole. The result showed that a sufficiently dense body becomes a black electromagnetic hole.

Key words - Gravitation, electromagnetic waves, photon and angular speed.



INTRODUCTION

The term black hole is of very recent origin. It was coined in 1969 by the American Scientist, John Wheeler, as a graphic description of an idea that goes back at least two hundred years, to a time when there were two theories of light, and the discovery by Loemer that the light travels at a finite speed meant that gravity might have an important effect.

On this assumption, a Cambridge don, John Michell, wrote a paper in 1783 in the Philosophical Transactions of the Royal Society of London in which he pointed out that a star that was sufficiently massive and compact would have such a strong gravitational field that light could not escape: any light emitted from the surface of the star would be dragged back by the stars gravitational attraction before it could get very far.

Although we would not be able to see them because light from them would not reach us, we would still feel their gravitational attraction. Such objects are what we now call black electromagnetic holes.

THE PRINCIPLES AND POSTULATES OF GENERAL MASS AND GRAVITATION

The existing theories of gravitation are based on the strong or semi-strong equivalence principles. The New theory of General Gravitation is based on the Principle of General Mass and General Inertia. The New Theory of General mass is stated as (Howusu, 1993)

“To the degree of accuracy of measurement available,

increases with velocity \vec{u} according to the relation

$$m(\vec{u}) = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}} m_0 \quad (1)$$

where c is the velocity of light in vacuo.”

Furthermore, “...in terms of the general mass m , and linear velocity, we shall define, as the most natural extension of Newton’s definition, the general linear momentum \vec{p}_g of a particle as follows:

$$\vec{p}_g = m\vec{u} \quad (2)$$

It follows that in all frames the general law of motion of a body should be defined as

$$\frac{d}{dt} \vec{p}_g = \vec{F}^e \quad (3)$$

Which is called the general law of motion, where \vec{F}^e is the external force acting on the particle.

To make this theory as open as possible, an arbitrary speed is associated with propagation of gravitational effect in vacuo and the law is stated as follows:

“ Let (\vec{r}', t') be an arbitrary point within a distribution of general mass particle occupy θ a region R of space-time. Let ρ be the general mass density at a point (\vec{r}, t) of space-time, a general gravitational potential Φ_g given by

$$\Phi_g(\vec{r}, t) = -G \int_{\Delta'} \frac{\rho(\vec{r}', t') d\tau'}{|\vec{r} - \vec{r}'|} \quad (4)$$

where G is the universal constant and the integration is over all points of space-time within τ' such that

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the mass m of a body of non-zero rest mass m_0

$$t - t' = \frac{|\vec{r} - \vec{r}'|}{c_g^2} \dots$$

Furthermore, considering the reciprocal effect of the action of the general gravitational field on all moving particles, it is formulated as the natural extension of Newton's law for the action of the universal gravitational field as follows

"Let m be the general mass of a particle Instantaneously at the point (\vec{r}, t) of Space-time, where there is a general Gravitational potential, Φ_g . Then the particle Instantaneously possesses a general gravitational potential energy U_g given by $U_g(\vec{r}, t) = m(\vec{r}, t) \Phi_g(\vec{r}, t)$ (5)

And experiences a general force, \vec{F}_g given by $\vec{F}_g(\vec{r}, t) = m(\vec{r}, t) \Delta \Phi_g(\vec{r}, t)$ (6)

The general gravitational force field is conservative by this definition.

A PHOTON IN A GENERAL GRAVITATIONAL FIELD

So far we have applied our postulates and laws of general mechanics and gravitation to particles and bodies with non-zero rest masses. In this section we shall study their consequences as applied to the behavior of the photon is a particle with the following properties.

- a) The rest mass of the photon is zero $m_0 = 0$ (7)
- b) When in motion a photon always has the linear speed c in all inertial reference frames $|u| = c$ (8)

- c) When in motion with an instantaneous frequency v, possesses a linear momentum \vec{p} given by

$$|p| = \frac{hv}{c} \dots (9)$$

Where h is Planc's constant.

- d) When in motion a photon with an instantaneous frequency v possesses an amount of energy given by

$$E = hv \dots (10)$$

and from a well-known mass-energy equivalence relation

$$E = mc^2 \dots (11)$$

The principle of general mass is extended to a photon as follows:

"when in motion a photon with an

Instantaneous frequency v possesses a General given by

$$m(v) = \frac{hv(r,t)}{c^2} \dots (12)$$

Now since a photon has a general mass, it follows that a photon moving \vec{u} with an instantaneous frequency v and linear velocity \vec{u} will possess a general momentum given by

$$\vec{p}_g(\vec{u}) = \frac{hv}{c^2} \vec{u} \dots (13)$$

Also, in accordance with the general mechanics, a photon will have a general law in all inertial reference frame given by

$$\frac{d}{dt} \vec{p}_g = \vec{F}_g \dots (14)$$

Considering a photon starting from a point (r_0, t_0) and moving in a general gravitational field having potential Φ_g such that it is instantaneously of the position (\vec{r}, t) in space-time with a velocity \vec{u} and using the conservation of mechanical energy in a general gravitational field, it can be shown that the general equation of motion for a photon is given by-

$$\Delta \Phi_g(\vec{r}, t) = \frac{d}{dt} \vec{u}(\vec{r}, t) - \left\{ \left[1 + \frac{1}{c^2} \Phi_g(r, t) \right]^{-1} \frac{d}{dt} \Phi_g(r, t) \right\} \frac{\vec{u}(\vec{r}, t)}{c^2} \dots (15)$$

This is the general equation of motion for a photon in a general gravitational field according to the new theories of general mechanics and gravitation.

EQUATION OF MOTION

Consider a photon moving in the gravitational field of a stationary homogeneous spherical body of radius R and rest mass M0. Let S be the reference frame whose origin O coincides with the centre of the body. Thus, in spherical coordinates $\vec{r} = \vec{r}(r, \theta, \phi)$ of S, the gravitational potential Φ_g , in the region exterior to the sphere is given by

$$\Phi_g'(r, t) = -\frac{a_0}{r} \dots (16)$$

where $a_0 = GM$ (17)

and G is the universal gravitational constant.

Also, in spherical coordinates

$$\vec{u}(r, t) = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\hat{\phi} \dots (18)$$

and

$$\vec{a}(r, t) = \frac{d\vec{u}(r, t)}{dt} \dots (19)$$

Hence, from (16),(18) and (19) and taking the photon to be moving in the plane $\theta = \frac{\pi}{2}$, for the sake of mathematical convenience, it follows that the component of the general

mechanical equations of motion for a photon in the gravitational field are given by

$$-\frac{a_0}{r} = \ddot{r} - r\dot{\theta}^2 - \frac{a_0}{c^2 r^2} \left(1 - \frac{a_0}{c^2 r}\right)^{-1} \dot{r}^2 \quad (20)$$

and

$$0 = r\ddot{\theta} + 2\dot{r}\dot{\theta} - \frac{a_0}{c^2 r} \left(1 - \frac{a_0}{c^2 r}\right) \dot{r}\dot{\theta} \quad (21)$$

such that the linear speed of the photon in the equatorial plane is given by

$$u^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad (22)$$

ANGULAR EQUATION

Equation (21) can also be written as:

$$\frac{\ddot{\theta}}{\dot{\theta}} + \frac{2\dot{r}}{r} - \frac{a_0}{c^2 r^2} \left(1 - \frac{a_0}{c^2 r}\right) \dot{r} \quad (23)$$

which integrates exactly to give

$$\dot{\theta} = \frac{\ell}{r^2} \left(1 - \frac{a_0}{c^2 r}\right) \quad (24)$$

where 1 is a constant of motion corresponding to the angular momentum per unit mass of the photon. This is the general mechanical angular speed of the photon in the equilateral plane.

RADIAL EQUATION

Our mechanical radial equation for the photon in the equatorial plane (20) may be written as

$$\ddot{r} - \frac{a_0}{c^2 r^2} \left(1 - \frac{a_0}{c^2 r}\right)^{-1} \dot{r}^2 = -\frac{a_0}{r^2} + r\dot{\theta}^2 \quad (25)$$

Now, substituting the angular speed of the photon (24) we have

$$\ddot{r} - \frac{a_0}{c^2 r^2} \left(1 - \frac{a_0}{c^2 r}\right)^{-1} \dot{r}^2 = -\frac{a_0}{r^2} + \frac{\ell^2}{r^3} \left(1 - \frac{a_0}{c^2 r}\right)^2 \quad (26)$$

This is our mechanical radial equation of motion for the photon in the equatorial plane.

Now by the transformation

$$W(r) = \dot{r}(r) \quad (27)$$

it follows that

$$\ddot{r}(r) = W \frac{dW}{dr}$$

Hence radial equation of motion (26) transform as follows

$$0 = \frac{dW}{dr} W - \frac{a_0}{c^2 r^2} \left(1 - \frac{a_0}{c^2 r}\right)^{-1} W^2 + \left[\frac{a_0}{r^2} + \frac{\ell^2}{r^3} \left(1 - \frac{a_0}{c^2 r}\right)^2 \right] \quad \dots(28)$$

This is Bernoulli's differential equation. Therefore, by the transformation

$$Z(r) = W^2(r) \quad (29)$$

Equation (28) reduces to first order differential equation

$$\frac{dZ}{dr} + p(r)Z(r) = Q(r) \quad (30)$$

where

$$p(r) = -\frac{2a_0}{c^2 r^2} \left(1 - \frac{a_0}{c^2 r}\right)^{-1} \quad (31)$$

and

$$Q(r) = -2 \left[\frac{a_0}{r^2} - \frac{\ell^2}{r^3} \left(1 - \frac{a_0}{c^2 r}\right)^2 \right] \quad (32)$$

The Solution of (30) is given by

$$\dot{r}^2 = Z(r) = \left(1 - \frac{a_0}{c^2 r}\right)^2 \left\{ A + 2c^2 \left(1 - \frac{a_0}{c^2 r}\right)^{-1} \frac{\ell^2}{r^2} \right\} \quad \dots(33)$$

But from (22) and (24)

$$u^2 = \dot{r}^2 + r^2 \dot{\theta}^2 = \dot{r}^2 + \frac{\ell^2}{r^2} \left(1 - \frac{a_0}{c^2 r}\right)^2 \quad (34)$$

Therefore, substituting (33) into (34) we have

$$u^2 = \left(1 - \frac{a_0}{c^2 r}\right)^2 \left[A + 2c^2 \left(1 - \frac{a_0}{c^2 r}\right)^{-1} \right] \quad (35)$$

where A is an arbitrary constant.

For a photon emitted at an infinite distance away from the body, we have the condition

$$\dot{r} = c \text{ at } r \rightarrow \infty \quad (36)$$

from which it follows that

$$A = -c^2 \quad (37)$$

Therefore, mechanical linear speed of the photon in the equatorial plane in terms of the radial coordinate is given as

$$u^2 = \left(1 - \frac{a_0}{c^2 r}\right) \left(1 + \frac{a_0}{c^2 r}\right) c^2 \quad (38)$$

Or equivalently,

$$u^2 = \left(1 - \frac{a^2_0}{c^4 r^2}\right) c^2 \quad (39)$$

BLACK ELECTROMAGNETIC HOLES

It follows from our mechanical linear speed of a photon in the equatorial plane in terms of the radial coordinates (39) that if the body is sufficiently massive or

dense, then there may exist our radius $r_H \square R$ such that

$$\frac{a_0}{c^2 r_H} = 1 \quad (40)$$

At which the linear speed of photon vanishes,

$$u = 0 \quad (41)$$

Physically, it means that:

A photon approaching the body from infinity is brought to a permanent rest at the radius r_H or a photon may not be emitted at the radius r_H .

Moreover, at a radius r such that

$$R \leq r < r_H \quad (42)$$

in the gravitational field of the body

$$\frac{a_0}{c^2 r} > 1$$

Implying that the linear speed of the photon is purely imaginary.

A body for which there exists our radius r_H is our black electromagnetic hole.

CONCLUSION

The mechanical equation of motion for a photon moving in an arbitrary gravitational field was derived and solved. It is shown from the result that a sufficiently dense body becomes a black electromagnetic hole.

REFERENCES

- [1] Gupta R.D. (1980): Mathematical physics; Vikas publishing home, London 157-167.
- [2] Howusu, S.X.K. (1993): The Natural Philosophy of Classical Mechanics, 58pp.
- [3] Izam, M.M. et al (2003): Effect of Temperature on the Gravitation of diatomic molecules- Science Forum: Journal of Pure and Applied Sciences Vol 6(1) 104-108.
- [4] Moller C.(1982): The Theory of Relativity, Oxford Clarendon Press, London 163-184
- [5] Moore, P.N.(1980): The Guinness Book of Astronomy- Facts and Feats, London, Guinness, 102pp.
- [6] Rindler W (1977): Essential Relativity, New York Springer Verlag Publishers. 1-9.
- [7] Weinberg: S.W. (1972): Gravitation and Cosmology; Principle and Applications of the General Theory of Relativity, New York. John Willey and Sons 175-210.